Modeling Left Ventricle Pumping

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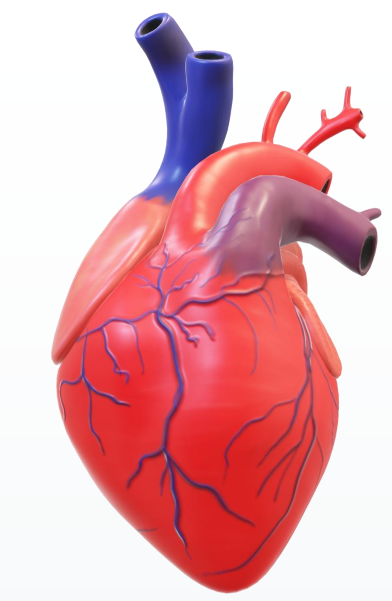


Abstract

The goal of this project was to create a model of a cardiac muscle strip contraction and then use that model to describe the left ventricle as a pump. Cardiac muscle has complex mechanical characteristics, which include both active, elastic nonlinear and passive, time-dependent varying linearity properties. These active and passive properties were built into the muscle strip contraction model, thereby defining a new constitutive relation for muscle. Utilizing LaPlace's Law, muscle force was related to generated left ventricular blood pressure for the assumed geometry of a cylinder. Human anatomical data were used to estimate the approximate number of muscle fibers in parallel necessary to generate realistic left ventricular pressure magnitudes. The model was then coupled to an arterial load model and pressures, flows, and volumes in the left ventricle and systemic arteries in the cardiovascular system were calculated for an ejecting heartbeat. Electrical circuit theory was used to solve the electrical analog of the arterial system, giving aortic pressure, ventricular pressure, ventricular outflow, and ventricular volume. Results compare favorably to physiological measurements of pressure in the normal human heart.

Background

$$f(t) = \frac{(1 - e^{-(\frac{t}{\tau_c})^{\alpha}})e^{-(\frac{t-t_b}{\tau_r})^{\alpha}}}{(1 - e^{-(\frac{t_p}{\tau_c})^{\alpha}})e^{-(\frac{t_p-t_b}{\tau_r})^{\alpha}}}$$

Equation 1: time course activation of force generation

$$f_m(t, l_m) = a(l_m - b)^2 + (cl_m - d)f(t)$$

Equation 2: generalized force generator equation

$$E_m(t, l_m) = 2a(l_m - b) + cf(t)$$

Equation 3: muscle elastance as a function of time and muscle length

$$l_m = (\frac{f_m}{f} + d)\frac{1}{c}$$

Equation 4: muscle length calculator based on force generator equation

Cardiac muscle includes active and passive properties to model force over the time of contraction. Equations 1-4 are derived from muscle mechanics and display how the left ventricle produces force and the subsequent elastic qualities, as well as in terms of muscle fiber length.

Constant	Value [units]
a	1.861 [mN/mm^2]
b	7.956 [mm]
c	19.68 [mN/mm]
d	158.1 [mN]
τ_c	0.152 [s]
t_p	0.36 [s]
τ_r	0.24 [s]
α	2

Methodology

$$\sigma = \frac{P * r}{t} = \frac{F}{A}$$

Equation 5: ventricle wall stress (LaPlace's Law)

$$P = \frac{f_m\,(r_o - r_i)}{\frac{\pi}{4}(r_o{}^2 - r_i{}^2)r_i}$$

Equation 6: pressure formula using generalized force generator

Z_0 = characteristic impedance
R_s = periplural resistance
C_s = arterial compliance
Q = flow
V = volume
P = pressure
S(subscript) = systolic
A(subscript) = aortic

$$P_v = \frac{(f_m \times l_m)}{V_v}$$

Equation 7: ventricle pressure formula using generalized force generator and length model

$$P_s = \frac{V_s}{C_s}$$

Equation 8: systolic pressure based on volume and compliance

$$Q_v = \frac{P_v - P_s}{Z_0}$$

Equation 9: ventricle flow rate from pressure and impedance

$$Q_s = \frac{P_s}{R_s}$$

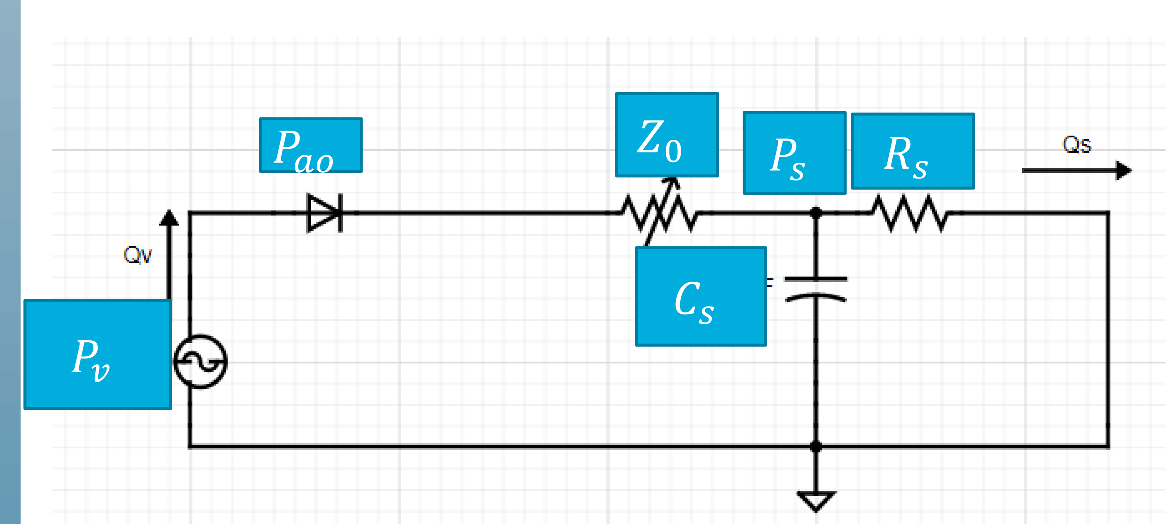
Equation 10: systolic flow based on pressure and resistance

$$V_v = A \times l_m$$

Equation 11: ventricle volume based on area and muscle length

$$\frac{dV_v}{dt} = dt(Q_v - Q_s)$$

Equation 12: rate of change in systolic volume based on change in flow



The mathematical computations involved in the modeling of the left ventricle pumping are shown above. While implementing the mechanics from Equations 1-4, the cardiac model can be derived. Equations 5 and 6 model ventricle wall stress and the pressure placed on cardiac muscle during heart contraction and relaxation. Equations 7-12 depict the circuit model and are based off circuit theory laws to represent blood flow through the heart.

Results

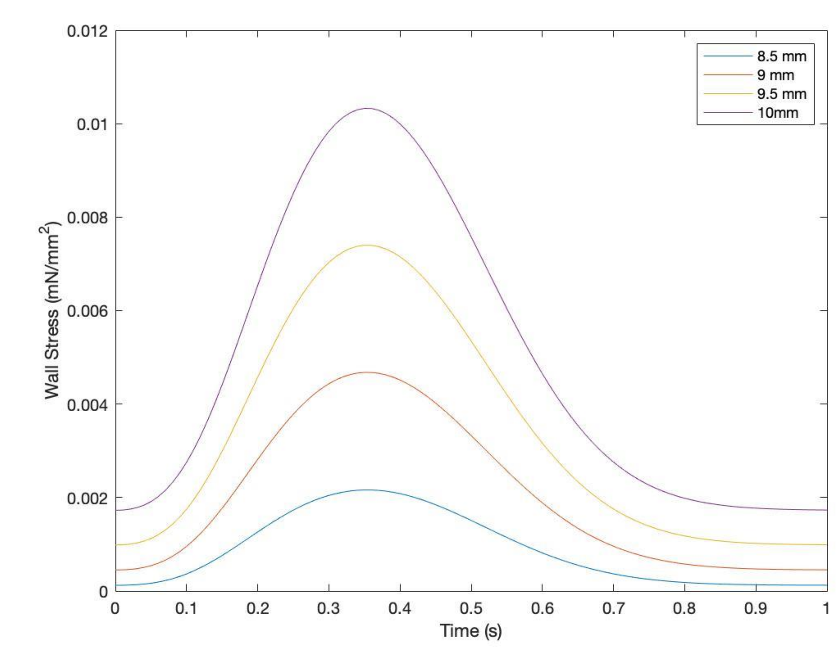


Figure 1: The ventricle wall stress for different lengths of muscle is shown above.

The wall stress on the left ventricle is derived through Law of Laplace. These stress values based on different lengths will be used in modeling total pressure the left ventricle experiences.

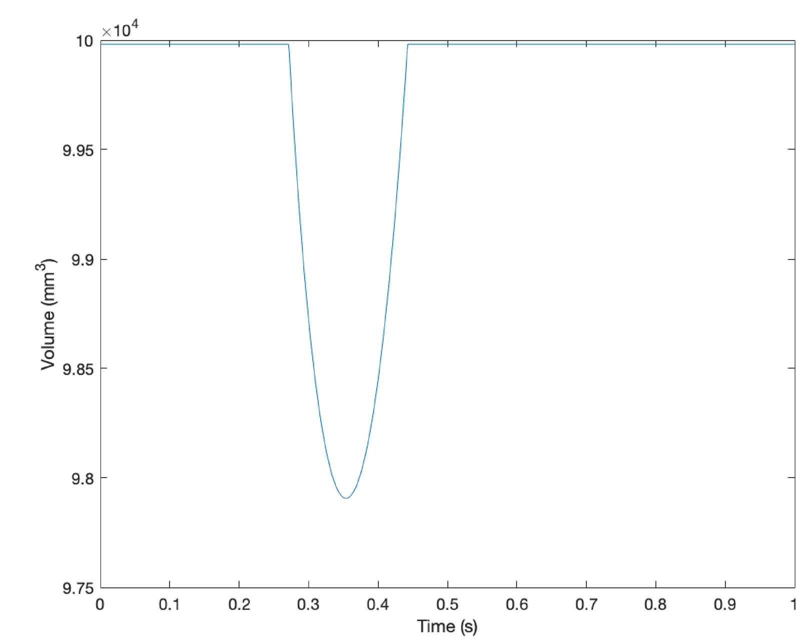


Figure 2: The change in blood volume during a heartbeat is shown above.

The volume of the left ventricle changes as the heart expands and contracts during each beat. This volume change is used in modeling pressure, flow, and arterial load changes.

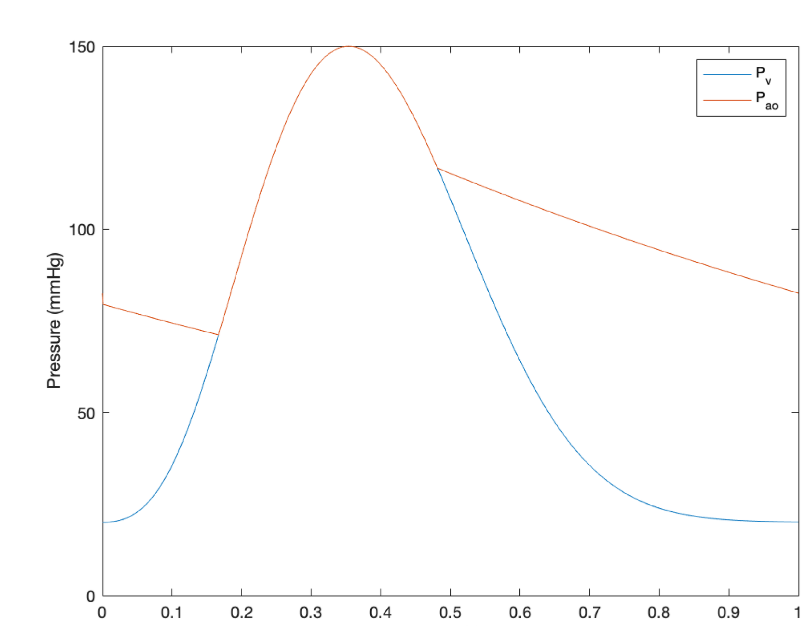


Figure 3: The left ventricular pressure and the aortic pressure during one heartbeat.

Conclusion

Throughout this design process, the fundamentals of muscular length, force, and elasticity were discovered to be crucial properties of cardiac contraction. The modeling of the left ventricle will further the understanding of the force, volume, and flow relationship of heart muscle. Simulation of heart diseases such as hypertension (increased blood pressure) and cardiomyopathy (thickening of heart muscle wall known as myocardium) can be properly researched with the software model. Increased cardiac education and experimentation can further pharmaceutical research to battle a major cause of death across the world; heart disease. Along with medical research, athletic cardiac strain can be examined to increase longevity and performance. The model depicts how blood flow and wall stress interact when the heart contracts and relaxes, which affects the entire circulatory system,

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