Design of a Flywheel-Controlled Inverted Pendulum

Lucas Duros ‘19, Gordon Hyduke ‘19, Jack McInnis ‘19
Faculty Advisor: Prof. Kevin Huang

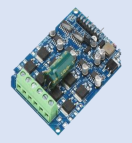
Abstract

An inverted pendulum is an unstable, nonlinear mechanical system which consists of a rigid pendulum with a center of mass located above its axis of rotation. This equilibrium is unstable, and any nonzero disturbance will cause it to fall due to gravity. The common control solution to the inverted pendulum problem consists of stabilization by translational motion of its base, as observed in ballbot and Segway-like devices. In contrast, this project explored an inverted pendulum control solution by means of attaching and actuating an inertial flywheel mounted to the rigid pendulum body. An input control torque applied to the flywheel by a brushed DC motor served to counteract any destabilizing torque. In order to employ linear control tools, a third-order linear state-space approximation of the system was calculated about the equilibrium. System identification unveiled physical motor parameters that prescribed a terminal motor voltage to generate the torque control input. With this setup, a modern linear control strategy, namely linear quadratic regulation (LQR), was employed to achieve stabilization of the nonlinear system. The LQR framework regulates the system to the zero state using state-feedback while simultaneously optimizing cost functions associated with system state parameters and control input. Simulated results using MATLAB/Simulink were compared to the results of the physical system.

Motivation

The solution to this problem is the first step towards the design of a **self-stabilizing motorcycle**. The self-stabilizing motorcycle provides a system analogous to training wheels for beginners as an alternative for safe learning.

Components

Component		Function	Cost
Arduino Mega		Microcontroller, implements logic of control system	Provided
Drok motor controller		Provides control input voltage to motor	\$25
Pololu motor with encoder		Determination of $\dot{\theta}$, applies torque to flywheel	\$60
Signwise encoder		Determination of $\phi, \dot{\phi}$	\$16
Total cost	-	-	**\$101**

Mathematical Model

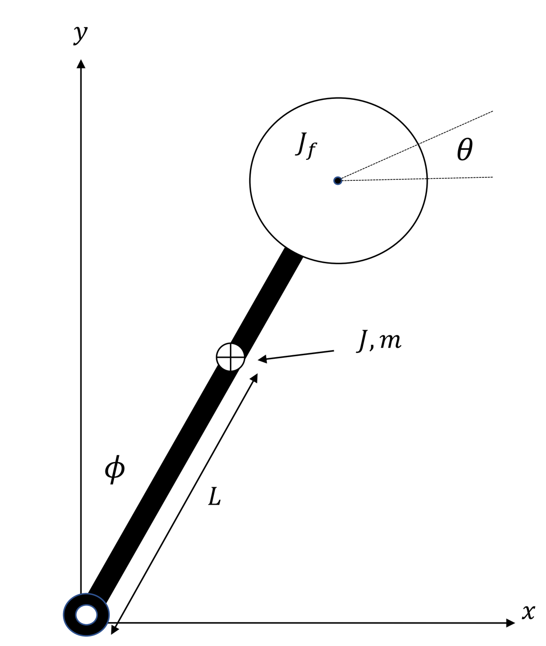


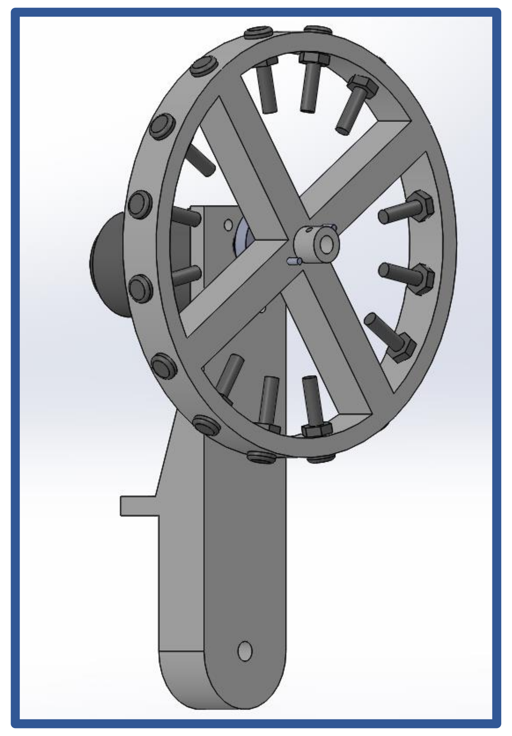
Figure 1: Diagram of pendulum-flywheel body with parameters labeled

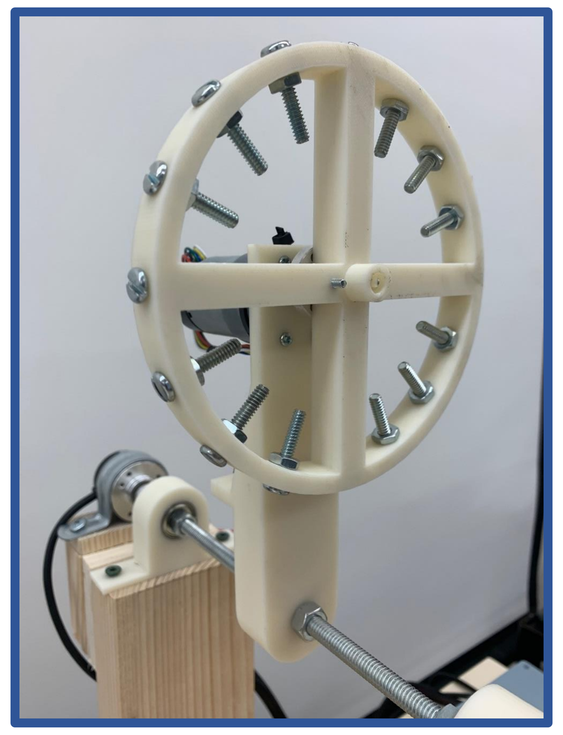
Governing equations of the linearized pendulum and motor were derived using the rotational form of Newton’s Second Law and Kirchoff’s Voltage Law. Matrix equations below show the complete pendulum model represented in state-space form with three state-variables ($\phi, \dot{\phi}, and\ \dot{\theta}$) and one control input (V_a). K_1 and K_2 are parameters of the motor which were experimentally found using system identification[3].

$$\begin{bmatrix}\dot{\phi}\\ \ddot{\phi}\\ \ddot{\theta}\end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ \frac{mgL}{J-J_f} & 0 & \frac{J_f * K_2}{J-J_f} \\ -\frac{mgL}{J-J_f} & 0 & -\frac{J * K_2}{J-J_f}\end{bmatrix}\begin{bmatrix}\phi\\ \dot{\phi}\\ \dot{\theta}\end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{J_f * K_1}{J-J_f} \\ \frac{J * K_1}{J-J_f}\end{bmatrix} V_a$$

$$\vec{x} = \begin{bmatrix}\phi\\ \dot{\phi}\\ \dot{\theta}\end{bmatrix}, \quad u = V_a\,, \quad \begin{bmatrix}\phi\\ \dot{\phi}\\ \dot{\theta}\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\phi\\ \dot{\phi}\\ \dot{\theta}\end{bmatrix} + \begin{bmatrix}0\\0\\0\end{bmatrix} V_a$$

Physical System Design





The pendulum body and flywheel rim were designed and modeled in SolidWorks then 3D printed using ABS plastic. Steel nuts and bolts were added to the outside of the rim of the flywheel to increase the moment of inertia of the flywheel. Next, the entire flywheel inverted pendulum was rendered in SolidWorks to find physical parameters such as location of center of mass (L) and moment of inertia of flywheel and entire body (J_f, J). Bearing supports were added to the tipping axle to reduce friction.

Control System Design

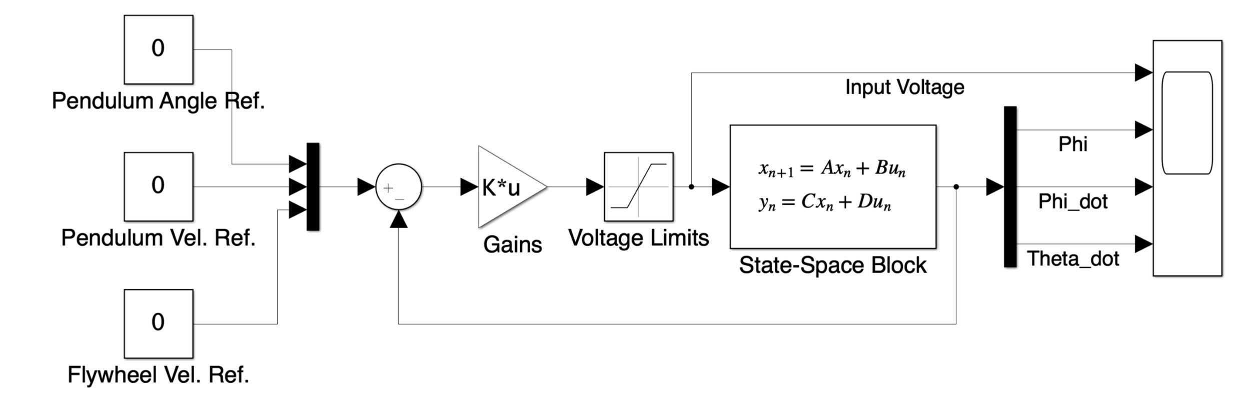


Figure 2: Control block diagram of state-feedback controller

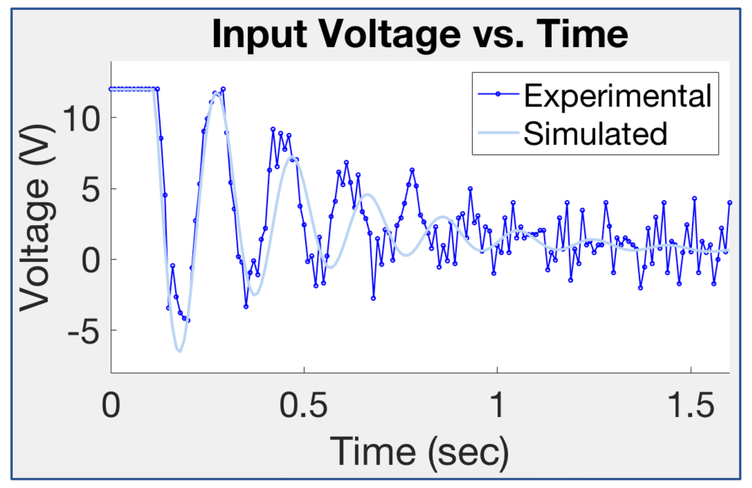
The linear-quadratic regulator (LQR) method was used to design the control system. LQR utilizes Q and R matrices to define a relative cost to each state-variable and control input. LQR minimizes a quadratic cost function of Q, R, x and u to solve for optimal gains.

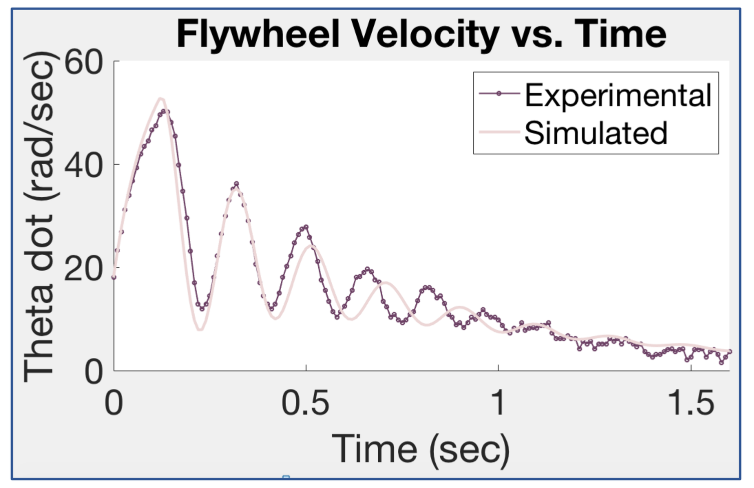
$$Q = \begin{bmatrix}100 & 0 & 0\\ 0 & 10 & 0\\ 0 & 0 & 50\end{bmatrix}, R = 200$$

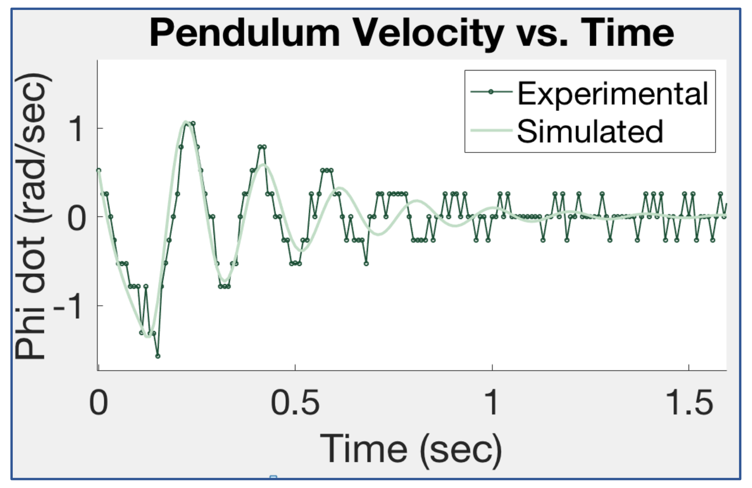
$$K = [-300 \quad -8.5 \quad -0.5] \quad (Gains)$$

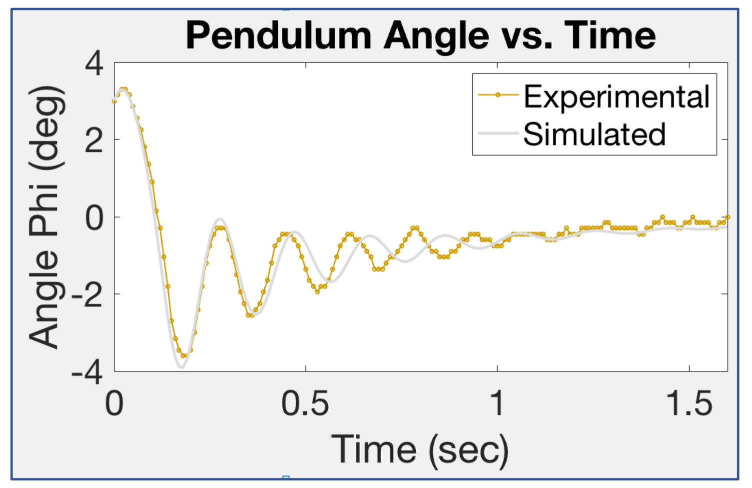
Results

Perturbations were applied to the stabilized physical pendulum system while the Arduino Mega collected data. Following the perturbation, the initial condition x(0) = [0.052 rad, 0.52 rad/s, 18.08 rad/s]^T was found and input to a Simulink model and simulated. Comparison of results are shown below. Simulated results are very similar to measured results. This helps confirm the validity of the mathematical model created.









References

[1] Olivares, Manuel, and Pedro Albertos. “Linear Control of the Flywheel Inverted Pendulum.” ISA Transactions, vol. 53, no. 5, 2014, pp. 1396–1403.

[2] Lin, J., Chen, S. Y., & Gau, W.-H. (2015). Design and implementation of a novel inertia flywheel pendulum mechatronic kit. Journal of Vibration and Control, 21(16), 3417–3430.

[3] Vasconcelos, José Roberto Canuto, et al. “Design And Control Of A Flywheel Inverted Pendulum System.” *Proceedings XXII Congresso Brasileiro De Automática*, 2018.

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Future Work

Swing up control of pendulum
- Add another control system for swinging up the pendulum to the unstable equilibrium position
- Switch back to controller designed in this project to keep the pendulum balanced

Move towards stabilizing motorcycle system
- Shift from a fixed axle pendulum to a standing body with one axis of rotation about its base
- Inertial mass unit (IMU) to determine angle of body
- Portable power supply – battery required