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CTL Fellows 2017-2018 Final Report
Applying Linear Algebra

The focus of my 2018-2019 CTL Fellow project was essentially the development of a course in the mathematics curriculum to address a few key issues that have been facing the mathematics department and its students for several years. As the title of my project indicates, this is a course in linear algebra with a bend towards applications and practical considerations. To understand the issues facing this course I will first give a brief background, followed by a summary of the course that I implemented, then conclude with some comments about things to change next year. Overall, the CTL Fellows program was very helpful. In addition to giving me the time necessary to work out details and ideas related to my course, it was very helpful to get advice from more senior faculty members and bounce ideas off of my peers, both from other scientific fields as well as the non-sciences.

Background —

While linear algebra plays a central role in the mathematics program at virtually every college or university throughout the world, it has historically held an additional role at Trinity. The mathematics major at Trinity requires two rigorous upper-level proof-based courses: Algebra (MATH 307) and Analysis (MATH 331). However, even after a strong performances in calculus and courses like differential equations, etc., students typically do not have the abstraction tools necessary to be successful in these courses. Therefore, our Linear Algebra course (MATH 228) plays double-duty, teaching linear algebra concepts, but also proof techniques and abstraction. However, we have begun to develop a number of additional proof-based courses to cover this base, and moreover we have more students now that seek more coursework in the applied side of mathematics. Therefore, I had two primary goals for my new course, entitled Applied Linear Algebra (MATH 229):

- Serve our growing population of mathematics majors that seek more experience in the area of applied mathematics, and
- Provide an better alternative for non-math majors (primarily in the other sciences) that seek a mathematics course outside of others such as Differential Equations, etc.

Thus, while the first goal seeks to address a local problem within the mathematics department, the second ensures that we better serve the larger Trinity community, hopefully facilitating connection between different disciplines.

First Implementation —

This past fall, fall 2017, was the first time the course was taught here at Trinity. Working up to the September kick-off I spent a large amount of time getting the course ready to go. Moreover, the format of the course is somewhat different than anything I have done in the past. The overall idea was to collect a series of applications from range of scientific disciplines, and use these as a foundation to motivate the mathematics. After working through the mathematical theory, the goal was then to loop back and solve a renal-world problem by putting theory in to practice. For example, I began the semester discussing medical imaging problems, and after a few weeks of theory, we looped back with a programming assignment where the students reconstructed an image using MATLAB in some data I provided. Other applications involve regression analysis, partial differential equations, population biology, and Google PageRank. (PageRank is my favorite of them all - I'd like to note that at the end of the semester I have the students actually program the PageRank algorithm and rank the webpages taken from the Notre-Dame web server.) Overall, I believe that the course was successful for a first iteration, although several changes are required which I'll discuss next. The course was surely challenging, as I kept the class going at a high level and fast pace.

Looking Forward —

I will have the opportunity to teach the course again for a second time this coming semester, fall 2018. A number of specific changes to the course will be in order. In particular, I will skip over a several applications and topics that I covered in the fall in favor of both (i) spending more time in areas that I believe I did not spend enough time on during the first time and (ii) adding a few new applications and topics. I've also decided to change the textbook to one that reflects these application and topic changes. Besides these changes, I will aim to maintain the structure of the course as a foundation of applications on which to base the mathematics. I believe that in the end the students appreciated the material more for having seen the applications from the outset. Perhaps having the knowledge that a specific theoretical concept is moving in the direction of an interesting problem keeps the students' attention.