Chapter 11

Dimensionless Invariants for Intentional Systems: Measuring the Fit of Vehicular Activities to Environmental Layout

Robert E. Shaw, Oded M. Flascher, and Endre E. Kadar
Intentional Dynamics Laboratory
Center for the Ecological Study of Perception and Action
The University of Connecticut

"When in use, a tool is a sort of extension of the hand, almost an attachment to it or a part of the user's own body, and thus is no longer a part of the environment of the user. But when not in use, the tool is simply a detached object of the environment... the boundary between the animal and the environment is not fixed at the surface of the skin but can shift."

—Gibson (1979, p. 41)

"The field of safe travel, it should be noted, is a spatial field but it is not fixed in physical space. The car is moving and the field moves with the car through space. Its point of reference is not the stationary objects of the environment, but the driver himself. It is not, however, merely the subjective experience of the driver. It exists objectively as the actual field within which the car can safely operate, whether the driver is aware of it. It shifts and changes continually, bending and twisting with the road, and also elongating or contracting, widening or narrowing, according as obstacles encroach upon it and limits its boundaries."

—Gibson & Crooks (1938, p. 454, emphasis in original)
11.0 Introduction

11.0.1 Aims and Motivations

Ecological psychology holds the belief that theory and basic research must ultimately aspire to practical applications. This follows directly from its primary aim to understand an actor's functional relationship to natural and manufactured environments. Although laboratory research is useful, it is no substitute for observations and measurements in the field. Even highly realistic simulations may be misleading. The safe flight of aircraft (Gibson, 1950) and the much earlier study of the field of safe travel for automobiles (Gibson & Crook, 1938; see above) presages the work we have undertaken here. The explicit aim of the evolving ecological approach is to develop methods of scientific investigation with ecological validity. Methods with ecological validity treat the organism and its environment as a system with variables defined on the ecological—environmental variables that make reference to and are scaled to the organism as a perceiver and actor in that environment. Methods are sought to explain, first, how actors achieve success on ecologically significant tasks performed in their natural environments and, second, to extend these methods to architectural and landscaped environments. This contrasts sharply with those methods that merely seek to understand how subjects attain statistical significance on arbitrary tasks performed in the laboratory. In this sense, the ecological approach aspires to treating the laboratory as an extension of the subjects' environce.

More than any other trait, the explicit commitment to strive for both ecological validity and ecological significance in one's research is the hallmark of an ecological psychologist. This commitment qualifies many to contribute as ecological psychologists, even though they call themselves by other names. One needs only share the belief that success on practical problems involving the perceptual control of action is an important, perhaps the most important, way to validate the consistency and the significance of one's theory and research. Thus, the attempt to wed ecological psychology with human factors and human engineering, as this volume tries to do, seems quite natural and overdue.

Our specific goal in this chapter is to present a new approach to the wheelchair navigation problem. How are wheelchair users able to select and follow the best route through cluttered architectural spaces, such as office and factory workplaces, or residential and public spaces? This chapter is, in part, a progress report of an ongoing research and theory on this topic.

Regarding this problem we have three aims—one practical and the others theoretical. One aim is to offer a practical solution to the problem of measuring the dynamic fit of active wheelchairs to their functional spaces under the constraints of a given navigation goal. The second aim is to place this problem under intentional dynamics—a general method of ecological psychology for modeling goal-directed activities. Although we use the wheelchair navigation problem as our focus, these methods and principles should apply to other activities as well. To meet these aims, we explore the use of dimensionless analysis—a mathematical engineering technique for finding dimensionless measures (called pi-numbers) of the similarity in the structure and functioning of systems which may appear quite different. We shall need to tailor this technique to certain prospective control problems. This entails our third aim, namely, to show how, in principle, dimensionless measures (pi-numbers) may allow us to compare the invariants of the information detected about the layout of the environment relative to an intended goal path to the invariant aspects of the control law that must be applied to navigate the path so as to attain that goal.

A caveat and apology are in order. Our aim is to introduce the steps required to arrive at this conclusion without developing or explaining in detail the mathematics involved. We recognize, therefore, that our explanations are somewhat opaque to the reader, but our aim is to point the way to a solution to this problem rather than to provide such a solution, for such details would go well beyond the space permitted for this chapter. We hope the reader nevertheless is stimulated to help develop the approach sketched here.

11.1 Task Similarity, Intentional Dynamics, and the Prospective Control Problem

The general navigation problem is a prospective control problem; it asks how perceptual information about a future state of affairs—an intended goal—can be used to control a current state of affairs (that is, the current forces) in order to reach that goal. More specifically, it asks how a rule for the perceptual control of action allows an actor to find his or her way through a cluttered environment over a preferred path to an intended goal. We explore the prospective control problem by focusing on adult human actors who locomote through architecutred environments by wheelchair. The task selected for them is a simple one—to pass without
to be found in discovery of the appropriate dimensionless numbers.

The transduction problem, which presupposes a solution to the
ecometrics scaling problem, is a problem for ecomechanics. Ecomechanics
asks how forceless information might be made efficacious in directing
control processes so that the actor might reach an intended goal state.
When classical mechanics is the study of laws governing motions
of inanimate bodies and biomechanics the study of laws governing
movements of biological systems, so ecomechanics is the study of laws
that govern actions (goal-directed movements) of agents (Shaw, 1987;
Kinsella-Shaw, 1988). Unlike the first two forms of mechanics,
the last one involves a special relationship holding between information
and control which goes beyond mere force or energy flow descriptions.
The details of this relationship have recently been spelled out as a theory
that information detection and energy control must be self-adjoint in the
sense of having mutual and reciprocal quantities (Kugler, &
Kinsella-Shaw, 1990; Shaw, Kadar, Sim, & Repperger, 1992).

Solving the scaling and transduction problems entails a new
approach to prospective control problems, one that is a hybridization of
physics, biology, and psychology. Attempts to develop such an
approach are being made. A branch of ecological psychology called
intentional dynamics subsumes ecometrics and ecomechanics. The
transduction problem of ecomechanics and the scaling problem of
ecometrics comprise dual aspects of the central problem of intentional
dynamics. Hence, the prospective control problem falls naturally under
this new discipline. (For an overview of intentional dynamics, see the
earlier studies mentioned as well as Kugler, & Shaw, 1990, and Kugler,
Only mention psychological processes of these categories in the
next section.

Before addressing the questions raised, we provide a statement of
the social motivation for this project and explain the practical
significance of its potential success.

11.2 Part I: A Problem with Ecological Significance
and a Method with Ecological Validity

11.2.1 Overcoming Barriers

An estimated half-million persons are added to the population of the
handicapped each year through illness or injury (Arthur, 1967). Of the
estimated 36 million Americans with disabilities (Disabled USA, 1984),
many achieve education and, later, employment or self-employment
because there are specially seated workplaces available for them in accessible facilities (for example, homes, schools, offices, and factories). Unfortunately, many other disabled Americans are excluded from these opportunities because standardized design guidelines fall short of the ideal and do not accommodate their particular disability. Current minimal standards sometimes fall short of the ideal for the disabled for two reasons: First, because the practices used to implement the standards may be underconstrained by the current specifications, or, second, because design tradeoffs are used to strike a practical and economical balance between the architectural dimensions needed to accommodate both the disabled and the nondisabled.

A significant reason for these problems is that, at present, it is impossible to measure the dynamic fit of goal-directed activities (e.g., moving wheelchairs) to the environments in which they must be performed. Consider the special case of designing environments for wheelchairs. Only recently have methods begun emerging to determine design guidelines for the fit of such activities performed from relatively stationary chairs and wheelchairs (see Dainoff, 1987, 1991a, 1991b; Abdel-Moty & Khalil, 1989). However, no significant work has yet been done on ways to measure dynamic maneuvers of wheelchairs over accessible routes through functional spaces toward goals. The proposed project offers a way to remedy this shortcoming.

In its specifications, the American National Standards Institute (ANSI, 1986) emphasizes the need to recognize that persons with disabilities that confine them to wheelchairs are no longer "average" persons. They are shorter and wider, rolling instead of walking, being unable to climb stairs. They require ramps or oversized elevators, require more space to turn around, and more clearance under tables and other equipment. Nor can they see over or cross over barriers that others might more easily. Moreover, they require, on the average, more time to egress through cluttered environments (such as furniture arrangements and milling crowds).

This list comprises but a few characteristics that distinguish the wheelchair-bound individual from ordinary individuals. Wheelchair users have fewer opportunities for actions than the nondisabled. Consequently, a question of great interest is how functional spaces for wheelchair users can be designed to afford freedom for them to act in ways comparable to nonwheelchair users. Consider other important characteristics of wheelchair users that make the designing of functional spaces explicitly for them even more critical.

11. DIMENSIONLESS INVARIANTS

11.2.2 Determining Dynamic Tolerances of Fit

The Uniform Federal Accessibility Standards (UFAS) sets the specification for a clear opening width for doorways. When the approach is head on, the recommendation is for widths of 36 in (91.5 cm), whereas turning a wheelchair to enter an opening requires greater clear widths. For most approaches, the addition of an inch lee-way on either side suffices—making for a minimum clear width of 32 in (81.5 cm). However, to accommodate the likelihood that control of straightline travel of the wheelchair will be imperfect adds at least 2 in tolerance. For instance, to accommodate traversing passageways that are more than 24 in (61 cm) long increases the minimum clear width to at least 36 in (91.5 cm) as compared to doorways with unrestricted approaches. For similar reasons, the minimal clear widths of checkout aisles in stores must also be 36 in (91.5 cm). However, the specifications must be even more generous in libraries. Because their greater length makes larger meanders from the straightline path even more likely, aisles between library bookcases require greater tolerances. In this case, the UFAS recommends a clear width of 42 in (106.5 cm) where possible. Where a wheelchair user might meet other wheelchair traffic or pedestrian traffic, then the clear width of passages must be adjusted to even greater tolerances (66 cm and 48 cm, respectively). Ultimately, our project aims at dynamic in-the-field testing to see if such standards remain realistic under a variety of wheelchair velocities, approach angles, and intentions.

These alterations to the specified clear widths of doorways or passageways, as a function of direction of approach, distance to traverse, or type of traffic, are only estimates. Dynamic measures might verify whether they are adequate adjustments to existing codes. Clearly, wheelchair speeds may increase when going down ramps, under emergency egresses, keeping up with traffic flow, and so forth. The possibility of varied speeds in approaching doorways or moving through passageways, therefore, requires dynamic measures to set their clear width tolerances. Wheelchair velocities may vary as a function of the layout of the environment, the circumstances, and the intentions of the wheelchair user.

Physical mechanics dictates that wheelchairs moving at even moderate speeds will have momentum characteristics that make them more demanding on space requirements than slow-moving ones. Momentum influences both maneuverability and control—sometimes in a positive way and sometimes in a negative way—depending on
circumstances. The formula for linear and angular momentum involves the multiplication of mass by velocity. For every unit of increase in speed or in the mass of the wheelchair user, there is a dramatic, multiplicative increase in the minimal requirements for stopping and turning. For this reason only dynamic measures can determine the limits on the safe and comfortable fit of the active wheelchair to architectural spaces.

In preparation for the rest of this chapter, it is well worth rereading the Gibson and Crooks' (1938) quotation given at the beginning of this chapter. Clearly, its full appreciation entails an innovative approach to accommodate the facts of dynamic measurement as discussed.

11.2.3 Limitations of Current Measurement Techniques

The Americans with Disability Act (ADA) provides an important incentive to actualizing the commitment of our society to allow every member of this community to participate, fully, in all aspects of life. Designing for architectural spaces that are accessible and usable by wheelchairs is a fundamental step toward equal rights. Legislation can do no better than to implement the best design guidelines that exist. If these are inadequate, then the resulting standards and codes will be equally wanting. In most cases, current standards have evolved from practical experience, legal precedent, and the intuition of experts. They have not been set nor verified by scientific methods. Without dynamic measurements there is no alternative to current practices. The simplicity of the proposed method can be better appreciated.

As observed earlier, the field needs dynamic measures of active wheelchairs to determine the safest, most comfortable, and efficient paths for traveling between points in the environment. Current methods may measure the total time to travel a fixed path, but not the selection of the path, nor the continuous time accumulated at each point along the path. A few experimental studies investigate wheelchair use in simulated environments. They use treadmills or dummies attached to wheelchairs, rather than human wheelchair users in actual workplace settings. None of the existing methods can make direct and continuous measurements of wheelchair use over paths that connect different areas in the workplace. We now appraise the existing techniques for dynamic investigations of this problem.

One might use aerial movies to measure wheelchair paths through natural environments. Unfortunately, aerial techniques prove impractical for several reasons: One must mount several cameras at heights greater than the ceilings usually available. Filming or taping the wheelchair's path requires an extremely wide angle lens that introduces gross distortions. This makes exact measurements problematic. Finally, the filmed or videotaped record from each camera must be digitized and their data somehow combined—a costly, time-consuming, and tedious job. Current automatic digitizing programs prove very expensive and are not, indeed cannot be, entirely automatic. Alternatively, automatic sonic digitized recordings are possible. This technique mounts sound emitters on the wheelchair and distributes an array of microphones widely over a prearranged path. As the wheelchair moves its location, speed, and direction are recorded. Such devices work best in limited spaces. They are also subject to serious problems of acoustic reflectance in nonstandardized environments. Neither audio nor video techniques work well in environments cluttered with furniture and other architectural barriers. These methods restrict the wheelchair routes to those within range of the camera or emitters. Consequently, they are unable to measure the user's natural preferences for routes that fall outside the predetermined set. These methods are thus less practical, more costly, and more restricted than the method to be proposed.

In short, we know of no current ecologically valid method for "in-the-field" measurements. Such a method is indispensable to the design of functional spaces in which wheelchair users perform daily a wide variety of diverse activities. A chief concern is that although ordinary environments may be sufficiently clear for walkers, they are usually not barrier free for wheelchair users. Hence, design criteria must differ for environments that allow these different modes of locomotion. Dynamic measurement of clear movement through such environments requires the development of new tools and techniques. We have built a prototype of a new measurement instrument and tested its feasibility which we describe next.

11.2.4 A New Technique for Measuring Dynamic Fit of Active Wheelchairs

For this project we have developed a unique tool and associated methods for determining the relevant variables for the fit of wheelchair activities following paths through functional spaces. Following Gibson and Crooks (1938), one of our aims is to discover how users perceptually select fields of comfortable (safe and efficient) travel
within fields of possible travel. A current series of experiments are designed to uncover the informational basis and the stable styles of control by which wheelchair users navigate successfully through doorways and passageways. In the heart of this research is a typical wheelchair that has been modified to gather data online, while running diverse routes through such environments. A prototype of the computerized wheelchair has been built and tested. These exploratory experiments are described later. Here we use the computerized wheelchair to examine a range of doorway and passageway width tolerances under a variety of experimental conditions with different velocities, widths, distances, and intentions. Our ultimate aim is to discover the relevant information variables, control parameters, and values or goals, whose interrelationships define the dynamic field that moving wheelchair users carry with themselves as they move about (Gibson & Crooks, 1938). For convenience, we might call these three kinds of parameters—observables, controllables, and values—the affordables\(^1\) of the task situation.

This prototype computerized wheelchair has an on-board, laptop computer that reads data from optically encoded accelerometers. The accelerometers are attached to a pair of measurement wheels that have been added to the undercarriage of a standard wheelchair between the large wheels by which the vehicle is steered and driven. This instrumentation allows the wheelchair's changing locations, orientations, and velocities to be automatically recorded at each point along the route traversed (within tolerances of approximately 2 mm for each 100 cm of lineal distance traveled). The existence of this first prototype demonstrates the feasibility of the concept, the soundness of

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\(^1\)Gibson (1979) introduces the term affordance for those objective properties of environments which, taken in reference to an actor, provide opportunities for that actor's actions (i.e., goal-directed behaviors). It is one thing, however, to have an opportunity for action and another to have the means to seize that opportunity; hence, the necessary distinction between affordances as opportunities to be seized and effectivities as the means by which such opportunities may be seized. Similarly, it is still another thing for an actor to value one opportunity more than others so that it and not they is intended and acted on—that is, to select, on a given occasion, one affordance as being more valuable than another; hence, the need for the term valuable to pick out the intended affordance. For these reasons, we introduce the term affordables to refer to the component parameters (observables, controllables, valuables) by which affordances, intentions that select them, and the effectivities that realize them, may be conveniently referred to under a common rubric.
control required for goal-directed actions to be successful.

11.3 Part II: Theoretical Background: Dynamic Fit as an Ecometric Problem

In this section we introduce some of the basic concepts that underlie ecometrics and ecomechanics. Such concepts will be involved in any attempt to solve the scaling and transduction problems associated with actors (e.g., wheelchair users) navigating successful goal paths.

Consider, first, some properties an environmental situation must have to afford an actor navigating through a cluttered environment: The medium that surrounds the actor must permit freedom of movement, and the actor—who may be a walker, flyer, swimmer, or vehicle user—must maintain mechanical contact with a surface of support. For swimming and flying creatures, the medium and surface of support are the same—water and air, respectively. For arboreal and land creatures, the medium is air or water, and the surface of support is tree, rock, ground, floor, stairs, or sidewalk. If the actor is neither arboreal nor aquatic, say a human, horse, or dog, then the surface must be sufficiently rigid to support the actor’s weight, have sufficient friction to allow traction, and be sufficiently level to prevent falling over. Environmental properties that afford opportunities for actions—that is, goal-directed activities—are called affordances (Gibson, 1979).

Examples of environmental affordance properties are the graspability afforded by certain objects, the supportability afforded by certain surfaces, and the edibility afforded by certain substances. In general, properties of objects or surfaces count as affordances if they provide appropriate structural and informational support for the action capabilities of properly attuned actors, that is, actors who have the means, opportunity, and motivation to carry out the relevant actions. Hence the definition of an affordance necessarily implicates corresponding action skills, called effectivities (Shaw & Turvey, 1981; Turvey & Shaw, 1979). Such effectivities determine whether a specific class of actors, for whom information specifying the relevant affordance property is available, can use that information to realize that affordance property, that is, to guide its behavior successfully toward an intended goal.

Table 11.1 shows examples of how affordances, effectivities, and actions have an underlying dimension of abstract similarity. Affordances and effectivities are functionally defined and functionally similar under the action of a sufficiently skilled organism relative to appropriate environmental structures. The affordance refers to the scale-dependent aspects of the physical situation that support an opportunity for a definite action; effectivities refer to the commensurate means available to the actor for realizing that definite action in that given situation. The actor’s intention to act toward realization of that affordance goal in the given situation provides the motive to act. As in a court of law, when all three conditions—means, motive, and opportunity—are met, then we may conclude that the agent has a realistic intention to act. Under this interpretation, an action is necessarily goal-directed and intentional, involving the goal-specific affordance and the intention-specific effectivity.

**TABLE 11.1: The Interrelationship of Affordances, Effectivities, and Goal-Directed Actions.**

<table>
<thead>
<tr>
<th>AFFORDANCE OF E</th>
<th>EFFECTIVITY OF O</th>
<th>ACTION OF O ON E</th>
</tr>
</thead>
<tbody>
<tr>
<td>(opportunity for action)</td>
<td>(means for acting)</td>
<td>(realizing intended affordance)</td>
</tr>
<tr>
<td>grasbable</td>
<td>able to grasp</td>
<td>O grasping E</td>
</tr>
<tr>
<td>climable</td>
<td>able to climb</td>
<td>O climbing E</td>
</tr>
<tr>
<td>catchable</td>
<td>able to catch</td>
<td>O catching E</td>
</tr>
<tr>
<td>passability</td>
<td>able to pass through</td>
<td>O passing through E</td>
</tr>
<tr>
<td>sit-on-able</td>
<td>able to sit</td>
<td>O sitting on E</td>
</tr>
</tbody>
</table>

A wheelchair is a vehicular tool, that is, a tool that aids locomotion. Hence, whatever similarities carry over from affordance information to effectivity control must do so through the tool that interfaces the actor-as-perceiver with the environment-as-perceived and acted on. But what happens to the affordance description of the environment and the effectivity description of the actor when tools as objects and tools as functions enter the story? We address this important question next.
11.3.1 Tools as Ecological Interfaces

Tools enhance, extend, or restore the action or perception capabilities of humans or animals. They may assist in manipulation, locomotion, or exploration. Tools may extend existing capabilities or restore lost functions either partially or completely. Tools may be simple machines (e.g., levers, ramps, pulleys, springs) or complex tools (e.g., trucks, bulldozers, automatic assembly lines, refineries) that amplify the capacity for work, or they may be devices that amplify information detection (e.g., microscopes, telescopes, sonar, radio, television) or amplify its usage (e.g., computers, libraries). Currently, it is still an open question whether computational tools may also amplify intelligence. They surely carry out, in many ways, the tasks that would otherwise take an intelligent person to do. In short, tools may focus or extend effective capabilities, thereby providing greater access to affordances in the environment and, consequently, new opportunities for action.

Tools may also serve a prosthetic function by restoring the effectiveness of lost or infirmed sensory abilities (e.g., eyeglasses or hearing aids), or they may restore lost or infirmed action capabilities. Some action prostheses restore manipulatory abilities, such as the artificial hand or arm; others restore lost or infirmed locomotory abilities, such as crutches, walkers, artificial legs, and wheelchairs. Manually driven or motor-driven wheelchairs fall into the category of vehicular tools. Little if any work, to our knowledge, has been done on the formal description of such tools and how they alter the dynamic fit of the disabled user to that environment in which they are typically used. Moreover, achieving empirical validation of the dynamic fit of vehicular tools might be furthered by considering how people navigate other vehicular tools, such as automobiles. We consider such comparisons later. A natural question is how does an object with a tool function relate to the affordances of the environment and the effectiveness of the actor? Does the tool act as a mediator between the environment’s affordances and the actor’s effectiveness. Or, if the tool does not sit in the seam between the environment and the actor, does it belong more to one component of the ecosystem than the other?

Figure 11.1 shows the relationship that a tool might have to the user and the user’s environment in which it is applied. Notice that the tool might be treated as a mediating device which belongs neither to the organism nor to the environment, but interfaces the two. This interpretation seems to overly complicate the relationship between organisms and their environments and to potentially destroy the

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Theoretical balance that is needed to keep affordances and effectiveness mutual and reciprocal (i.e., mathematically dual) as illustrated in Table 11.1. This theoretical balance is desirable because it allows the relationship between information and control to remain epistemically direct for all the reasons that we have discussed at length elsewhere (Shaw & Turvey, 1981; Turvey & Shaw, 1979; Turvey, Shaw, Reed, & Mace, 1981). Mathematically, this directness in the coupling of the components of the ecosystem allows us to use some powerful theorems from (self-adjoint) information/control theory that permit an economy of description for modeling psychological ecosystems, affordances and effectiveness, information and control, and the perceiving-acting cycle that would not otherwise be possible (Shaw et al., 1992; Shaw, Kugler, & Kinsella-Shaw, 1990). There seems to be another alternative which keeps intact our fundamental assumptions.

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(a) Tool as a mediator between O and E

(b) Tool as functional extension of O

(c) Tool as functional extension of E

Figure 11.1. Possible ways to partition the role of a tool in an ecosystem: (a) as a mediator, where E↔T↔O; (b) or as an extension of environmental affordances, where (E↔T)↔O; or (c) or as an extension of an actor’s effectiveness, where E↔(T↔O).
We prefer to distinguish between tools-as-objects and tools-as-functions. As objects, tools must be treated as contributing to the affordance structure of the environment. That is, before the tool is used, like any object it has its own affordances, thereby inviting certain actions. But tools-in-use are no longer just objects but play an intrinsic role in extending, replacing, or restoring the user’s repertoire of effectivities—that is, the actor’s capabilities for realizing affordances of the environment. Tools, therefore, have a dual function—as objects within the environment’s affordance structure and as components within the actor’s effectivity system. Through the dual function, a tool both scales and transduces the information detected and the control executed in the course of the task involving the tool use.

For instance, on the ground a steel rod may afford grasping, lifting, and wielding. But once grasped and properly braced under a boulder, the rod becomes a lever and is part of the effectivity for leveraging the rod causing it to roll down a hill. Hence, to call this object a rod is to distinguish its role in the affordance structure of the environment, whereas to call it a lever is to distinguish its function as an integral part of an effectivity system. There is clearly a logical distinction between the affordance structure of an object that invites a goal-directed function and the effectivity, or goal-directed function, it invites. These concepts should not be confused. The former refers to the environment as a source of goals that an organism might achieve, whereas the latter refers to the organism as a source of intentions that achievement of environmental goals might satisfy. Hence, there is no ambiguity introduced by dually partitioning the tool, first, as a part of the functionally defined environment with affordance properties and, then, as part of the functionally defined organism with effectivity properties, as shown in Figure 11.1b and 11.1c.

Consider the methodological benefits of recognizing the dual functional roles that tools play: As objects they have affordances like any other, but as tools they engage effectivities, thereby permitting actors to perform tool-specific, goal-directed functions. Thus, a tool function of an object contributes, in a unique way, to the affordance-effectivity compatibility underlying an actor’s fit to its environment. Furthermore, and here is the main point, because tool use requires a high degree of coordination between perception and action, tools focus behaviors. When theoretical description and empirical study might be intractable at the microscale level of the enormous number of degrees of freedom exhibited by neuromuscular acts, both theory and experiment become more tractable when only the macroscale degrees of freedom of

the tool using system is considered.

This suggests a way that one might develop a manageable inventory of solutions to the degrees of freedom recognized by Bernstein (1967). The scientific study of tool functions involved in manipulatory and locomotor acts might identify more tractable analogues of their corresponding, more complicated, unaided goal-directed actions. A study of these “free-action” analogues might provide a theoretical basis for better understanding the underlying effectivities that underwrite them.

If tools promote the fit of actors to their environments in ways more visible than tool-independent behaviors, then studying activities involving wheelchairs may help us understand the coordination requirements for successful locomotor navigation in general. For describing the behavioral interface of the wheelchair to the floor surface is much simpler than describing the free actions of a walker. Of course, so far as specifics are concerned, the study of one is not a substitute for the other. Even so, as related locomotor cases, they are equally interesting examples of how similar systems may exhibit intentional dynamics under different navigational demands.

For these reasons, it would be important to have a way of measuring the degree to which tools promote the common fit of actors to the task environment, as compared to the same actors without the tool. It seems reasonable to assume that use of the same tools by different actors performing the same tasks increases the task constraints so that they become dynamically more similar as intentional systems. We turn next to a discussion of various ways in which systems (actors plus tools) might be similar in performing goal-directed behaviors.

11.3.2 Similarity across Physical Systems

Assume there are two systems, each with the same tool interface with the environment but different masses. If the tools and tasks are similar, then their behaviors should be similar—differing by only a scale factor reflecting their different masses. For instance, imagine two wheelchair users of different weights who must select efficient paths for navigating through a room cluttered with furniture so as to exit through a doorway. If they travel at the same speed (are kinematically similar), the heavier actor (greater mass) will require larger turning radii because he or she will have the greater momentum (mass x velocity). In spite of the fact that the two cannot travel the same curvilinear paths at the same
velocity, there are still numerous ways that they may perform their tasks similarly. They may cover similar distances to get to the doorway, take similar amount of time to do so, go in similar directions, use similar work (i.e., proportional to their masses), and so forth.

Table 11.2 shows the dimensions involved in the various kinds of similarity that one system might have to another with respect to the paths they follow in navigating through an environment. (Similarity means alike up to a scale factor). Geometric, kinematic, and kinetic path similarity hold for systems that have proportional relationships among state variables involving the dimensions of L, LT, LT³, or DT³, respectively. Likewise, work, impulse, and torque similarity hold if the two systems exhibit proportional path functions (integrals) involving these same dimensions in which each dimension is multiplied by force (i.e., \( fL \), \( fT \), \( fT^3 \), respectively). Specifically, geometric path similarity means the two systems cover similar distances between homologous environmental locations; kinematic path similarity means the two systems arrive at homologous places on a similar temporal schedule; and kinetic path similarity means the two systems apply similar forces to move over similar distances (similar work forces), times (similar impulse forces), and directions (similar torque forces).

Table 11.2: Various Kinds of Physical Similarity Two Systems Might Exhibit.

<table>
<thead>
<tr>
<th>Similarity over</th>
<th>length (L)</th>
<th>time (T)</th>
<th>angle (D)</th>
<th>force (F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>distance</td>
<td>√</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>time</td>
<td></td>
<td>√</td>
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<td>direction</td>
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<td>√</td>
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<td>work</td>
<td>√</td>
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<td>impulse</td>
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<tr>
<td>torque</td>
<td>√</td>
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Perceptual similarity involves a common proportionality defined over information variables detected by the systems, whereas actional similarity involves a common proportionality over control variables used by the two systems to produce their respective behaviors. Finally, intentional similarity means the two systems share similar goals (i.e., target parameters, manner parameters, or both). When the goalpaths pursued by the two systems are similar, then we say that they are similar with respect to their intentional dynamics. We now describe how this works.

Table 11.3 indicates the intentional dynamical similarities that two systems might exhibit in their goal-directed performance, in which the physical similarities defined in Table 11.2 refer to measures indicating how the path is generated from the initial condition (start-up) to some final condition. This path may or may not reach the target nor do so in the manner of approach intended. By contrast, the intentional dynamics similarities, defined in Table 11.3, refer to measures indicating how the path ought to be generated to conform to an intended final condition (reaching the intended target in the manner intended). Hence, Table 11.2 refers to measures of the causal generation of the mechanical path indifferent to the goalpath, whereas Table 11.3 refers to measures of the intentional control required to generate the eomechanical path, that is, the intended goalpath. The former is a "blind push from behind" by

Table 11.3: Various Kinds of Intentional Dynamical Similarity Two Systems Might Exhibit.

<table>
<thead>
<tr>
<th>type of similarity between systems</th>
<th>distance to contact (δ)</th>
<th>time to contact (τ)</th>
<th>direction to contact (γ)</th>
<th>work to contact (F x δ)</th>
<th>impulse to contact (F x τ)</th>
<th>torque to contact (F x γ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>target</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td>manner</td>
<td></td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>√</td>
</tr>
<tr>
<td>goal-path</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>√</td>
</tr>
</tbody>
</table>
forces determined retrospectively by physical law as a function of the
past states of the system; the latter is a 'directed pull from in front'
determined prospectively by the actor's control law being provided with
(perceptual) information about how the observed future states of the
system reckon with those intended. For example, time as a variable for
physical dynamics tells us how much time has elapsed since the system
left its initial start-up state given where it is; whereas time-to-contact, as a
variable for intentional dynamics, tells how much time should elapse
before the system reaches its intended final state, given where it is. A
similar prospective interpretation should be given to the other "contact"
variables.

The importance of extending physical similarity theory to include
intentional dynamical similarity is to allow measures of the similarity
between intentionally driven systems analogous to the similarity
measures possible for causally driven systems. Dimensionless analysis,
an outgrowth of dimensional analysis and similarity theory, provides a
way to construct measures of physical similarity between the observed
characteristics of two systems as well as between a single system's
observed and intended characteristics. Both of these similarities will be
of interest to comparing systems with both physical as well as
intentional dynamics. The general nature of these measures is discussed
next.

11.3.3 Dimensionless Ratios as Measures of Similarity

Dimensionless analysis developed from dimensional analysis: the
classic parameter approach introduced by Buckingham (1914) and
Rayleigh (1915), systematically developed by Bridgman (1922),
reformulated by Drobot (1954) to recognize dimensionless functions,
and used by Schuring (1977) to emphasize law-based similarities.
Dimensionless analysis serves two important functions for model
construction and theory. First, it puts a limiting framework around the
model to be constructed. This framework is based on the necessity of
any functional relation to remain invariant if the units are changed.
A second important function capitalizes on the fact that any physical
system can be analyzed into functions of a limited number of

2The notion of eco-work and affordance-effectivity duality as dimensionless
quantities, were introduced in a series of lectures on eometrics given in the late
1970’s at the University of Connecticut by R. Shaw, first reported in Warren &
Shaw (1981), and given experimental application by Warren (1982).

dimensionally fundamental variables that define its equation of state.
The number of independent dimensionless groups, as proven under the
π-theorem, is equal to the difference between the number of variables
that make them up and the number of dimensions involved (Ipsen,
1960). We explore these two functions of dimensionless analysis in the
context of the tasks involving the perceptual control of actions required
to perform wheelchair navigation tasks and other locomotory tasks.
Precedents for using such an approach in psychology have been set by
Warren and Shaw (1981), Kugler, Kelso, and Turvey (1980, 1982), and
especially applied to ecological psychology by Shaw and Warren as
reported in part by Warren (1982).

The physical dimensions of interest, as discussed earlier, are those of
length (L), time (T), direction (D) and force (F). The L, T, D, F system of
variables are fundamental in that they can be used to define any other
variables, called derived variables. Using these fundamental dimensions,
the respective functions describing a pair of systems can be compared so
as to reveal any similarities that might exist. Such similarities may exist
at various levels of analysis with respect to the state equations of the
systems.

In order for a comparison of the equations for two systems to be
legitimate, however, certain formal criteria must be satisfied. One
important criterion, discovered by Fourier in 1822, is that such equations
be dimensionally homogeneous, that is, be expressed in the same
dimensions. (Consequently, a solution to the scaling and transduction
problems introduced earlier depends on this property holding between
the equations describing the information and those describing the
control as exhibited by the perceiving-acting cycle involved in a given
task.) On initial inspection, however, two equations may not appear to
satisfy dimensional homogeneity because they may be expressed in
different derived variables whose symbols are quite distinct. However,
by reducing derived variables to the fundamental dimensions of F, L, D,
T (or the corresponding fundamental dimensions, F, δ, τ, γ, for contact
variables), it is possible to determine if two systems satisfy this
homogeneity property. For example, in Newton’s famous law (stated in
a form to emphasize its dimensional analysis), F = MA, although mass
and acceleration appear as derived variables, they can be expressed as a
function of the fundamental dimensions so that A = LT⁻² and M =
F(LT²)⁻¹.

The most important progress in the study of similarity among
systems resulted in the development of the so-called π-theorem
(Buckingham, 1914), one of the great achievements of similarity theory.
This beautiful theorem asserts that any analysis of a physical phenomenon in dimensional terms can always be reduced to a simpler functional relationship among dimensionless variables (Drobot, 1954; Rosen, 1978; Stahl, 1963). An appreciation of this theorem is necessary if one is to understand how systems too complicated to be described in terms of mathematically closed formulae (e.g., differential equations) might nevertheless be compared in terms of more abstract levels of similarity. The worth of this approach was summarized by Stahl (1963) as follows:

"It is reiterated that similarity criteria may always be obtained in a simple manner from governing differential equations, when such equations are available. When clear differential formulations are not at hand much progress can be made by the study of simplified model formulations of the problem, followed by an apparent dimensional ‘integration’ which gives the pertinent similarity criteria without actually performing the numerical integration process. In other cases there may be no differential equation which appears appropriate and one can choose similarity criteria on the basis of prior experience and direct manipulation of dimensional variables." (p. 369)

This is not the place to go into details regarding this approach, but fortunately several lucid accounts are available that the interested reader might consult (Birkhoff, 1960; Duncan, 1953; Johnstone & Thrting, 1957; Langhaar, 1967; Sedov, 1959; and in the area of similarity of biological systems, see especially, Günther, 1975; Rosen, 1978; Stahl, 1963). To give the spirit of dimensional analysis and similarity theory, and to show how dimensionless numbers naturally arise in the former and have application in the latter, consider a classical example—the case of the generalized Reynolds number.

An Example: The Reynolds Number. In 1638 Galileo made initial contributions to similarity theory by noting that static objects, such as pillars, tree trunks, and animal’s legs, should scale area of support as a function of volume of the mass supported. Newton went still further by recognizing that laws of nature should be formulated so as to be independent of certain dimensions, such as the overall size of objects. Finally, similarity theory moved to a new plateau of abstraction and generality when it was recognized that one might avoid specific dimensions altogether by composing variables from the ratios of two variables which were not just identical in dimensionality, but whose dimensions canceled. Such variables, when evaluated, came to be known as “dimensionless numbers”—the most famous of which is the Reynolds number.

Specifically, a variable is dimensionless if it is formed by a ratio of two quantities measured in units of the same dimension so that these units cancel out of the numerator and the denominator, leaving a pure ratio. A so-called n-number is a numerical evaluation of a dimensionless variable. The numbers receive their name from the fact that the well-known geometric ratio \( n = 3.1415 \) is itself a dimensionless number, one achieved by dividing the circumference of a circle by its radius, where both are measured in the same units. Hence, the units cancel leaving a dimensionless n-number. Quantities in which this is so are said to satisfy the property of dimensional homogeneity.

The Reynolds number, perhaps the most famous example, derives from the Navier–Stokes equation which describes the behavior of fluids. In this context, the Reynolds number expresses abstractly the ratio of inertial forces to viscous forces acting on a small volume of fluid. To be more specific, the Reynolds number is composed in the following way (Here, for convenience, mass, \( M \), is used as a fundamental rather than a derived dimension):

\[
\frac{\nu L^2}{\eta}
\]

in which \( \nu \) is velocity (LT\(^{-1}\)), \( L \) is characteristic length, \( \eta \) is viscosity (ML\(^{-1}\)T\(^{-1}\)), and \( \rho \) is density (ML\(^{-3}\)). If we substitute these fundamental dimensional quantities in this formula, it confirms that Reynolds number is dimensionless; namely,

\[
(LT^{-1})(ML^{-3})/(ML^{-1}T^{-1}) = LLLLMTL3MT = MTL3MT1T1L3 = [1].
\]

(Brackets indicate a dimensionless quantity rather than an integer.)

The use of dimensionless numbers in science is exemplified by applying the Reynolds number in biology. For instance, it has been shown that hemodynamical systems (blood flow) are similar across the circulatory systems of a variety of animals because they all share fundamental similarities with hydrodynamical systems in general (Stahl, 1963). In general, a rather large set of invariant dimensionless numbers have been found that seem to hold for complex multivariable biological systems. These numbers specify relationships of these otherwise diverse systems that are independent of specific parameters such as size. One theorist goes so far as to claim inductively that “behind the complexity and astonishing variety of forms and functions quantitative criteria of
similarity for all living systems have been disclosed" (Günther, 1975, p. 660).

11.3.4 Dimensionless Measures of Similarity Between Information and Control Processes

In the same spirit, the Gestalt psychologists claimed that abstract structure can be recognized, even though transposed from one situation to another (Koffka, 1935). Similarly, Gibson (1979) has spoken of invariant information as being both "timeless" and "formless," that is, as being independent of the specific parameters of a given environmental situation. Recent evidence suggests that information underlying perception of environmental structure must be very abstract. If what comes in on the perception side is to be of use to what goes out on the action side, then there must be a similarity of goal-specific information detected to the control which is guided by that information. For example, a cat must detect the target information, the direction and distance to be jumped, in order to land on a perch, say, the top of a fence. In the information picked up both from the environment and from its own body in that environment, there must be a reasonably precise specification of the forces required to aim itself and the impulse forces required to do the necessary work of transporting its body mass to the target. These general requirements for any successful action by any actor can be summarized by using the concept of affordables introduced earlier.

The observables present in the perceptual information specify the affordance goal to be selected (e.g., the fence's perchability). The affordance's observables are then transduced into ecologically scaled, biomechanical degrees of freedom, or controllables. If these action-control parameter values define a successful goal-directed function, or effectivity, for accomplishing the intended goal (e.g., jumping to the fence), then they comprise the set of values, or valuables, sought. More generally: If the mapping of the observables over the dually scaled controllables allows an actor to transduce its energy into the intended valuables, then an opportunity for action is successfully seized. This is the condition that allows the affordance and the corresponding effectivity to be duals, and as such it is guaranteed that the information and control are sufficiently similar so that the former can be both transduced and scaled into the latter.

Indeed, a target's observables must be sufficiently abstract to be invariently transposed from the particular form of information they must take to be detected to the particular form they must take to be the controllables that allow a successful action toward that target. This is the basic assumption of ecometrics (Shaw & Kinsella-Shaw, 1988). Mathematically, this assumption entails that a "flow" of information and the "flow" of control over the perceiving-acting cycle together define a dynamic invariant, or what physicists call a conserved quantity. A formal argument can be made that this generalized abstract quantity, called the total action potential (Appendix A), must be conserved if one is to explain how action skills acquired in one situation can generalize to other similar situations (see Shaw et al., 1990, for details).

Furthermore, the conserving of this total action potential whenever a goal-directed behavior is successful has been mathematically shown to entail that an inner (scalar) product invariant (Appendix B) must hold between the dynamic flow of goal-specific information and the dynamic flow of the control of goal-relevant energy expenditures (for details, see Shaw et al., 1992). In other words, the information detected over the path to a goal must determine the path over which the actor controls its manner of approach to the goal—otherwise, either the target moved toward is not the intended target or the manner of approach is not the one intended. In either case, if there is an error in information detection or control, the path traveled will not be the intended goal path.

Proving the existence of a conserved quantity, or dynamic invariant, as a condition for success of goal-directed behavior might be called the fundamental theorem of ecometrics. Showing empirically that this theorem does in fact hold in a variety of experimental contexts for different actors is very important for motivating research into systems that exhibit intentional dynamics, that is, goal-directed behaviors. For the theorem asserts that an abstract similarity must hold over the perceiving-acting cycle whenever an actor successfully uses perceptual information to achieve an intended action goal—a statement tantamount to the claim that a dimensionless quantity exists which is a kind of intentional dynamical invariant and may be constructed over the relevant task affordables. These are clearly π-numbers at the ecological scale rather than an arbitrary physical scale, as illustrated in the following example.

Consider two cats who satisfy their respective intentions equally well by jumping to the top of a fence. Let one cat be more massive than the other. If we construct the equations for each performance (analogous to Reynolds's equations for flow), the control and information variables should be similar (just as the inertial and viscous forces were similar). The pair of resulting ratios of these similar quantities will
produce a \( \pi \)-number that is the same for both cats, being indifferent to the difference in their masses. The total action for a given cat comprises certain functions of the information detection and energy control that maintain an invariant relationship just in case the cat's behavior is successful. When the two cats' behaviors satisfy similar goals, then other similarities should hold as well. For instance, the parabolic paths through the air to the fence top should be geometrically similar in shape and kinematically similar in the time to traverse the paths; likewise, the work-to-contact with the fence top should be kinetically similar, and so forth. The fact that they also have the same targets and similar manners of approach to those targets and hence similar goal paths should be reflected in the dimensionless ratio. Hence, the similarity of their goal-directed tasks with respect to both ordinary causal dynamics and intentional dynamics should be revealed by their respective \( \pi \)-numbers being identical. The degree to which these numbers are not identical provides a measure of the dissimilarity of their actions. This is the promise of dimensionless analysis that we wish to explore in tasks involving different actors performing the same or similar locomotory tasks.

Our research project, therefore, ultimately aims at determining the empirical validity of the fundamental theorem of ecometrics, as sketched earlier. As a means for exploring this thesis of intentional dynamics, there is no problem more scientifically interesting nor socially significant problem than wheelchair navigation. In the next section, we outline the method of attack on this problem and present some recent encouraging results. It should be clearly noted, however, at the outset that although these results obtained encourage the use of \( \pi \)-numbers, they also indicate that the current use has been too limited to solve the prospective control problem in its most general form (e.g., locomotory navigation toward a goal through a cluttered environment). Consequently, after reviewing the promising but limited results of current research by ourselves and others, we shall discuss the mathematical foundations of \( \pi \)-numbers. Our goal will be to suggest the ways of removing these limitations so that \( \pi \)-numbers might be generalized to measure similarity of any actions by systems operating at the ecological scale under the aegis of intentional dynamics.

11.4 Part III: Recent Investigations of the Fit of Actors to their Environments

11.4.1 \( \pi \)-numbers of Different Orders

Anthropometry is the study of the design of environments, furniture, tools, and equipment in units proportional to the intrinsic measurement of the human body (Panero & Zelnik, 1979). The architect Corbusier employed anthropometric principles, on a large scale, in the design of cities and buildings and, on a smaller scale, in the design of furniture and equipment. The famous Bauhaus initiated exploratory investigations into such design principles in the 1930s. Since then, the anthropometric approach has become an integral part of all design disciplines, especially architecture, human factors, and human engineering. Anthropometric \( \pi \)-numbers are those dimensionless ratios based on body-scaled measurements that play a role in the design and evaluation of environmental structures, furniture, tools, and equipment. Clearly, anthropometric \( \pi \)-numbers constitute an important class of geometric \( \pi \)-numbers. Very recently the empirical investigation into the validity of such geometric \( \pi \)-numbers has grown in popularity.

Specifically, in our laboratory and elsewhere, we use \( \pi \)-numbers to express similarities between energy control and information detection. As argued earlier, under the basic ecometric theorem of intentional dynamics, such similarities should exist whenever actors must make compensatory motor adjustments, given goal-specific information, to carry out successful goal-directed activities. Such compensatory motor adjustments, under the control of perceptual information, specify the actor's fit to the relevant environmental structures. This is the pragmatic meaning of the perceiving-acting cycle (Gibson, 1979).

A decade ago, as his dissertation, Warren carried out a now famous study exploring the kinetic basis for anthropometric (geometric) \( \pi \)-numbers (Warren, 1982, 1984). He showed that for people of different heights the perceived optimal stair design was specific to the person's individual height defined as a function of leg length. (Here the measure of optimal stair design is the riser-to-tread ratio taken relative to the climber's leg length.) Nevertheless, the ratio of leg dimensions to stair dimensions for all people (riser height in cm/leg length in cm = \( r/l \))—short and tall ones—were identical (\( r/l = \pi_{r/t} = .88 \)). These people then had to climb a variable motorized stairmill. The stairmill ran at speeds requiring a wide range of step frequencies (30 to 70 cycles/min), while the subjects climbed as comfortably as possible. Warren obtained an
important result. Subjects produced an invariant optimal work measure (\(\pi_a = .26\), as measured in calories/kg-cycle by oxygen utilization methods) only for those designs that matched the dimensionless number obtained from their original perceptual judgments.\(^3\) This research showed convincingly the existence of anthropometric \(\pi\)-numbers that hold predictably over both perceptual and action measures (\(\pi_a\) and \(\pi_p\)). How general are these \(\pi\)-numbers?

Kocza, Meeuwesen, and Cress (1992) investigated stair designs with maximum riser heights. They asked whether leg strength and hip-joint flexibility provide additional relevant constraints on both the perceptual judgment and action capability of subjects regarding such stairs. Thus, their work generalized Warren's conditions. Warren measured only anthropometric factors, omitting the effect of different dynamic conditions (such as different speeds) and used only young adults of the same approximate ages. Kocza et al. (1992), by contrast, showed that younger and older adults differ in the perception of maximum climable riser height. In other words, adults can accurately perceive the relative limitations and, presumably, the change in their action capabilities that aging brings. Specifically, their analysis showed that one's action capability (in stair climbing) is subject to multiple biomechanical factors (that is, kinetic \(\pi\)-numbers). These factors, taken together with anthropometric constraints, are better descriptions of the action capability than the anthropometric constraints (geometric \(\pi\)-

\(^3\)The action \(\pi_a\)-number, on first consideration, may not seem to be truly dimensionless as it must be in order to be a \(\pi\)-number since it involves a kinetic quantity (minimum energy expenditure per vertical meter measured in units of calories/kg-m) taken in reference to a geometric quantity (leg length). However, \(\pi_a\) is found by, first, plotting energy expenditure, \(E_d\), as a function of riser height, \(R\), and, then, taking \(1/R\), the riser height value at which \(E_d\) is minimal, as numerator and leg length, \(L\), as denominator to obtain \(\pi_a = R_0/L = .26\) for all people regardless of size or age as the stair design is anthropometrically optimal for them. This was Warren's (1982) method. A more general method exists for computing \(\pi\)-numbers from dimensionalized measures that are dimensionally inhomogeneous. If one takes the measures in two different situations, as over learning trials for an individual, a test-retest, or over different individuals, dimensionless ratios can be constructed from measures that fail to satisfy the property of dimensional homogeneity. Let the two measures taken in the first situation be \(k_1\) and \(k_2\), dimensionalized as \(MLT\) and \(LT\), respectively; and where \(k_1\) and \(k_2\) are the same measures taken on a different occasion. We can then form a dimensionless quantity by taking their cross-ratio as follows: \(k_1MLT/k_2LT\).

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numbers) taken alone.

By using \(\pi\)-numbers, these experiments make an exciting discovery. They show that the control of energy expended for action (kinetic \(\pi\)-numbers) is formally similar to the information available through visual perception (geometric \(\pi\)-numbers). But where is the dimensional homogeneity required? It must be supplied in some way, for it seems that people can "see" the work to be done, the impulse forces to be scheduled, and the torques by which to direct their behaviors toward selected goals in the manner intended. Perhaps, the underlying geometric commensurability required for this to be so is a similarity relationship existing between something felt and something seen. Could it be the similarity between the "felt effort" experienced during the act of climbing the stairmill and the information detected in seeing the stair design? Is this not the source of the commensurability of haptic information with visual information required for their dimensional homogeneity? We return to this point later.

Other approaches have also successfully constructed \(\pi\)-numbers for different activities than stair climbability. Each of these is constructed from a ratio of an anthropometric variable with an environmental variable, defining what we call an ecological \(\pi\)-number. In principle, such ecological \(\pi\)-numbers may be anthropometrically based (geometrical \(\pi\)-numbers), biomechanically based (kinematic \(\pi\)-numbers), bioenergetically based (kinetic \(\pi\)-numbers), or intentionally dynamically based (ecological \(\pi\)-numbers). Researchers have identified a range of \(\pi\)-numbers in different scientific domains. In psychology, following Warren (1982, 1984), Mark (1987) found geometrical \(\pi\)-numbers, whereas Lee (1974) and Todd (1981) found kinematic ones. As pointed out, in at least one case these numbers have been validated both for perceptual judgment and for physiological measures of action under normal conditions. These findings suggest that the human perceptual system is a fine measuring device for different dimensions of locomotions, such as climbability (Warren), sittability (Mark), catchability (Todd), braking ability (Lee), and passability (Warren & Wang, 1987).

In addition, we have extended this methodology to wheelchair passability studies. We review these results next.

11.4.2 Wheelchair Passability Studies

With the exception of the work on driving (e.g., Gordon, 1966) or braking automobiles (Lee, 1976), or flying and landing of airplanes (e.g., Warren, 1991), research on the dynamic fit of actors to their