PHYS 231 **3. Take-home Exam**

due: May 2nd 2008 in my office

The rules of the game:

same as last time (if you're unsure look them up on the first take-home exam)

1. Throckmorton Strikes Again !

An ant with mass m is standing peacefully on top of a horizontal, stretched rope. The rope has mass per unit length μ and is under tension F. Without warning, cousin Throckmorton (oh, how I hate that little bastard...) starts a sinusoidal wave of wavelength λ propagating along the rope. The motion of the rope is in a vertical plane. What minimum wave amplitude will make the ant momentarily weightless. Assume that m is so small that the presence of the ant has no effect on the propagation of the wave.

2. Mass Spectrometers

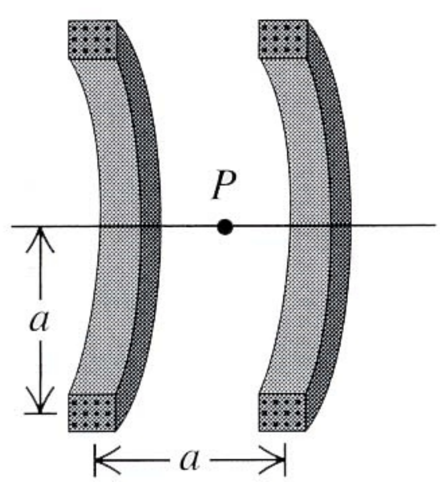
Climatologists can determine the past temperature of the earth by comparing the ratio of the isotope oxygen-18 to the isotope oxygen-16 in air trapped in ancient ice sheets such as those in Greenland. In one method for separating these isotopes, a sample containing both of them is first singly ionized (one electron removed) and then accelerated from rest through a potential difference V. This beam then enters a magnetic field B at right angles to the field and is bent into a quarter circle. A particle detector at the end of the path measures the amount of each isotope.

a) Derive an expression for separation $\Delta r = R_{18} - R_{16}$ of the two isotopes

b) If the magnetic field is 0.050T, what must be the accelerating potential V so that these two isotopes will be separated by 4.00 cm at the detector?

3. Helmholtz Coils

The figure below is a sectional view of two circular coils with radius *a*, each wound with N turns of wire carrying a current I, circulating in the same direction in both coils. The coils are separated by a distance *a* equal to their radii. In this configuration the coils are called Helmholtz coils; they produce a very uniform magnetic field in the region between them .



a) Derive the expression for the magnitude B of the magnetic field of a point on the axis a distance x to the right of point P, which is midway between the coils.

b) Graph B vs. x for x = 0 to x = *a*/2. Compare this graph to one for the magnetic field due a single coil with twice the windings, but otherwise identical characteristics, located at the position of the left-hand coil.

c) Calculate dB/dx and d^2B/dx^2 at point P (x = 0). Discuss how your results show that B is very uniform in the vicinity of P.

4. Motional emf's in Transportation

Airplanes and trains move through the earth's magnetic field at rather high speeds, so it is reasonable to wonder whether this field can have a reasonable effect on them. We shall use a typical value of 0.5 G (Gauss) for the earth's magnetic field.

a) The French TGV train reaches speeds of up to 180 mph and moves on tracks roughly 1.5 m apart. At top speed moving perpendicular to the earth's magnetic field, what potential difference is induced across the tracks as the wheels roll? Does this seem large enough to produce noticeable effects?

b) A Boeing 747-400 aircraft has a wingspan of 64.4 m and a cruising speed of 565 mph. If there is no wind blowing (so that this is also the plane's speed relative to the ground), what is the maximum potential difference that could be induced between the opposite tips of the wings? Does this seem large enough to cause problems with the plane?

5. Electromagnetic Waves in Conductors

Electromagnetic waves propagate much differently in conductors than they do in dielectrics or in vacuum. If the resistivity ρ of the conductor is sufficiently low (that is, if it is a sufficiently good conductor), the oscillating field of the wave gives rise to an oscillating conduction current that is much larger than the displacement current. In this case the wave equation for an electric field

$$\vec{E}(x,t) = E_y(x,t)\,\hat{j}$$

propagating in the +x direction within a conductor is

$$\frac{\partial^2 E_y(x,t)}{\partial x^2} = \frac{\mu}{\rho}\frac{\partial E_y(x,t)}{\partial t}$$

a) Show that

$$E_y(x,t) = E_{max} e^{-kx} \sin(k_c x - \omega t), \quad \text{with } k_c = \sqrt{\omega\mu/2\rho}$$

is a solution to this wave equation.

b) The exponential term shows that the wave decreases in amplitude as it propagates. Explain why this happens.

c) Show that the electric field amplitude decreases by a factor of 1/e in a distance $1/k_c$, and calculate this distance for a radio wave with $f = 1.0$ MHz in copper ($\rho = 1.73 \times 10^{-8}\ \Omega$ m, $\mu = \mu_0$)