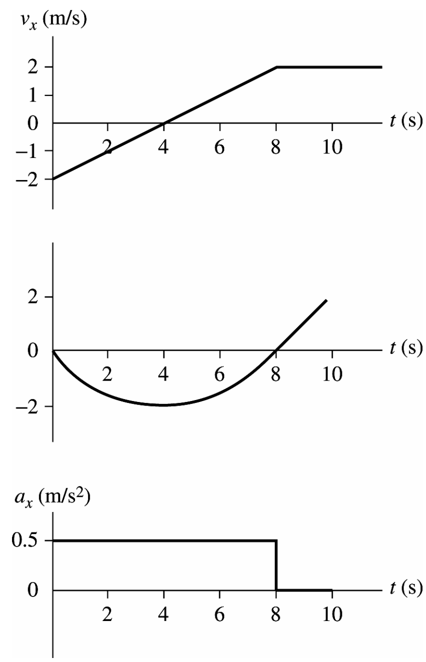
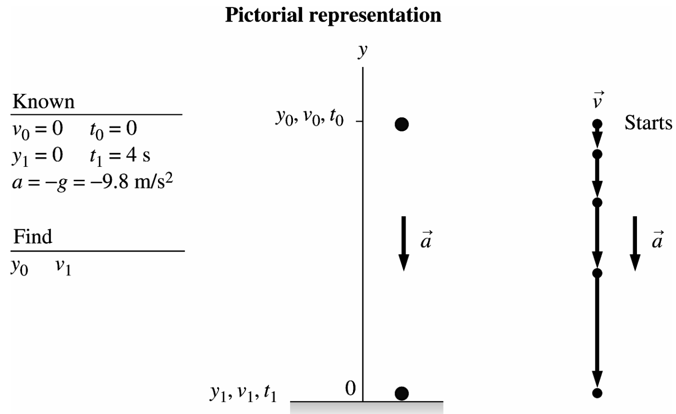
2.9. Visualize: Please refer to Figure EX2.9.

Solve: **(a)** The acceleration of the train at $t = 3.0$ s is the slope of the v vs t graph at $t = 3$ s. Thus $a = (2\text{ m/s} - (-2\text{ m/s}))/(8\text{ s}) = 0.5\text{ m/s}^2$.

(b)



2.15. Model: Represent the spherical drop of molten metal as a particle.
Visualize:



Solve: **(a)** The shot is in free fall, so we can use free fall kinematics with $a = -g$. The height must be such that the shot takes 4 s to fall, so we choose $t_1 = 4$ s. Then,

$$y_1 = y_0 + v_0(t_1 - t_0) - \frac{1}{2}g(t_1 - t_0)^2 \Rightarrow y_0 = \frac{1}{2}gt_1^2 = \frac{1}{2}(9.8 \text{ m/s}^2)(4 \text{ s})^2 = 78.4 \text{ m}$$

(b) The impact velocity is $v_1 = v_0 - g(t_1 - t_0) = -gt_1 = -39.2$ m/s.

Assess: Note the minus sign. The question asked for *velocity*, not speed, and the *y*-component of $\vec{v}$ is negative because the vector points downward.

2.21. Solve: **(a)** The position $t = 2$ s is $x_{2\text{ s}} = [2(2)^2 - 2 + 1]$ m $= 7$ m.

(b) The velocity is the derivative $v = dx/dt$ and the velocity at $t = 2$ s is calculated as follows:

$$v = (4t^2 - 1)\text{ m/s} \Rightarrow v_{2\text{ s}} = [4(2) - 1]\text{ m/s} = 7\text{ m/s}$$

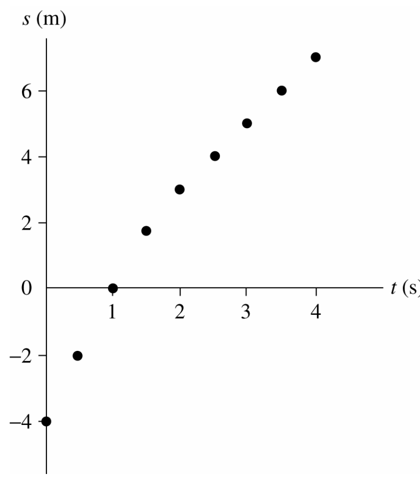
(c) The acceleration is the derivative $a = dv/dt$ and the acceleration at $t = 2$ s is calculated as follows:

$$a = (4)\text{ m/s}^2 \Rightarrow a_{2\text{ s}} = 4\text{ m/s}^2$$

2.24. Solve: (a)

Time (s)	Position (m)
0	–4
0.5	–2
1.0	0
1.5	1.75
2.0	3
2.5	4
3.0	5
3.5	6
4.0	7

(b)



(c) $\Delta s = s(\text{at } t = 1 \text{ s}) - s(\text{at } t = 0 \text{ s}) = 0 \text{ m} - (-4 \text{ m}) = 4 \text{ m}$.

(d) $\Delta s = s(\text{at } t = 4 \text{ s}) - s(\text{at } t = 2 \text{ s}) = 7 \text{ m} - 3 \text{ m} = 4 \text{ m}$.

(e) From $t = 0$ s to $t = 1$ s, $v_s = \Delta s/\Delta t = 4$ m/s.

(f) From $t = 2$ s to $t = 4$ s, $v_s = \Delta s/\Delta t = 2$ m/s.

(g) The average acceleration is

$$a = \frac{\Delta v}{\Delta t} = \frac{2 \text{ m/s} - 4 \text{ m/s}}{2 \text{ s} - 1 \text{ s}} = -2 \text{ m/s}^2$$

2.34. Solve: (a) The velocity is the integral of the acceleration.

$$v_{1x} = v_{0x} + \int_{t_0}^{t_1} a_x dt = 0 \text{ m/s} + \int_0^{t_1} (10 - t)\, dt = \left(10t - \tfrac{1}{2}t^2\right)\Big|_0^{t_1} = 10t_1 - \tfrac{1}{2}t_1^2$$

The velocity is zero when

$$v_{1x} = 0 \text{ m/s} = \left(10t_1 - \tfrac{1}{2}t_1^2\right) = \left(10 - \tfrac{1}{2}t_1\right) \times t_1$$

$$\Rightarrow t_1 = 0 \text{ s} \quad \text{or} \quad t_1 = 20 \text{ s}$$

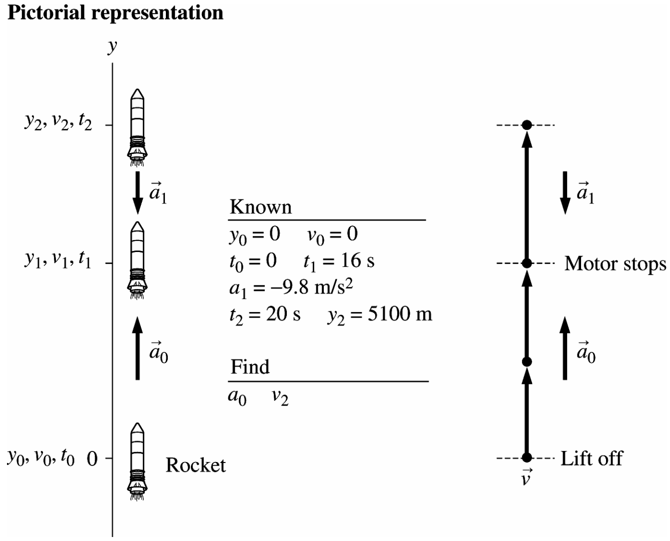
The first solution is the initial condition. Thus the particle's velocity is again 0 m/s at $t_1 = 20$ s.

(b) Position is the integral of the velocity. At $t_1 = 20$ s, and using $x_0 = 0$ m at $t_0 = 0$ s, the position is

$$x_1 = x_0 + \int_{t_0}^{t_1} v_x dt = 0 \text{ m} + \int_0^{20} \left(10t - \tfrac{1}{2}t^2\right) dt = 5t^2\Big|_0^{20} - \tfrac{1}{6}t^3\Big|_0^{20} = 667 \text{ m}$$

2.50. Model: We will model the rocket as a particle. Air resistance will be neglected.

Visualize:



Solve: **(a)** Using the constant-acceleration kinematic equations,

$$v_1 = v_0 + a_0(t_1 - t_0) = 0 \text{ m/s} + a_0(16 \text{ s} - 0 \text{ s}) = a_0(16 \text{ s})$$

$$y_1 = y_0 + v_0(t_1 - t_0) + \frac{1}{2}a_0(t_1 - t_0)^2 = \frac{1}{2}a_0(16 \text{ s} - 0 \text{ s})^2 = a_0(128 \text{ s}^2)$$

$$y_2 = y_1 + v_1(t_2 - t_1) + \frac{1}{2}a_1(t_2 - t_1)^2$$

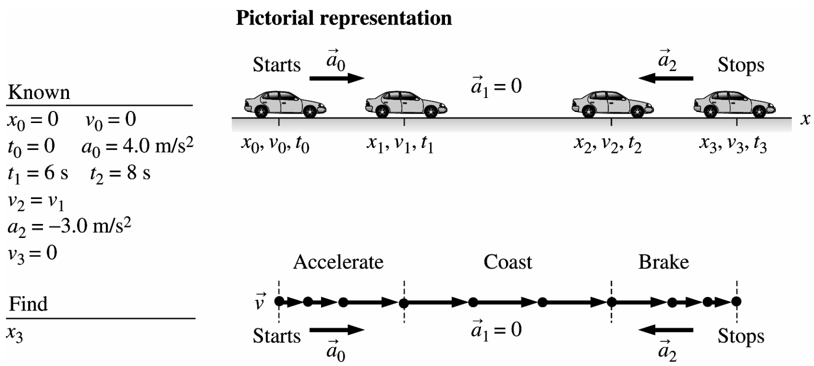
$$\Rightarrow 5100 \text{ m} = 128 \text{ s}^2 \ a_0 + 16 \text{ s } a_0(20 \text{ s} - 16 \text{ s}) + \frac{1}{2}(-9.8 \text{ m/s}^2)(20 \text{ s} - 16 \text{ s})^2 \Rightarrow a_0 = 27 \text{ m/s}^2$$

(b) The rocket's speed as it passes through a cloud 5100 m above the ground can be determined using the kinematic equation:

$$v_2 = v_1 + a_1(t_2 - t_1) = (16 \text{ s})a_0 + (-9.8 \text{ m/s}^2)(4 \text{ s}) = 4.0 \times 10^2 \text{ m/s}$$

Assess: 400 m/s ≈ 900 mph, which would be the final speed of a rocket that has been accelerating for 20 s at a rate of approximately 20 m/s^2 or 66 ft/s^2.

2.53. Model: The car is a particle moving under constant-acceleration kinematic equations.
Visualize:



Solve: This is a three-part problem. First the car accelerates, then it moves with a constant speed, and then it decelerates.
First, the car accelerates:

$$v_1 = v_0 + a_0(t_1 - t_0) = 0 \text{ m/s} + (4.0 \text{ m/s}^2)(6 \text{ s} - 0 \text{ s}) = 24 \text{ m/s}$$

$$x_1 = x_0 + v_0(t_1 - t_0) + \frac{1}{2}a_0(t_1 - t_0)^2 = 0 \text{ m} + \frac{1}{2}(4.0 \text{ m/s}^2)(6 \text{ s} - 0 \text{ s})^2 = 72 \text{ m}$$

Second, the car moves at v_1:

$$x_2 - x_1 = v_1(t_2 - t_1) + \frac{1}{2}a_1(t_2 - t_1)^2 = (24 \text{ m/s})(8 \text{ s} - 6 \text{ s}) + 0 \text{ m} = 48 \text{ m}$$

Third, the car decelerates:

$$v_3 = v_2 + a_2(t_3 - t_2) \Rightarrow 0 \text{ m/s} = 24 \text{ m/s} + (-3.0 \text{ m/s}^2)(t_3 - t_2) \Rightarrow (t_3 - t_2) = 8 \text{ s}$$

$$x_3 = x_2 + v_2(t_3 - t_2) + \frac{1}{2}a_2(t_3 - t_2)^2 \Rightarrow x_3 - x_2 = (24 \text{ m/s})(8 \text{ s}) + \frac{1}{2}(-3.0 \text{ m/s}^2)(8 \text{ s})^2 = 96 \text{ m}$$

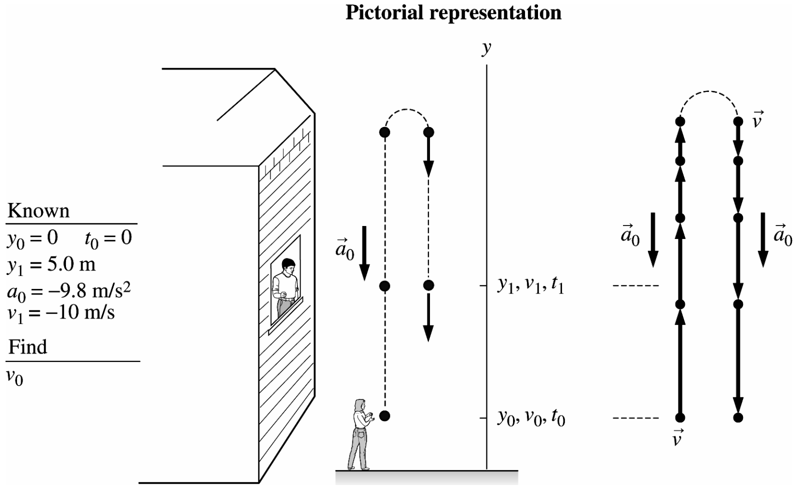
Thus, the total distance between stop signs is:

$$x_3 - x_0 = (x_3 - x_2) + (x_2 - x_1) + (x_1 - x_0) = 96 \text{ m} + 48\text{m} + 72 \text{ m} = 216 \text{ m}$$

Assess: A distance of approximately 600 ft in a time of around 10 s with an acceleration/deceleration of the order of 7 mph/s is reasonable.

2.63. Model: The ball is a particle that exhibits freely falling motion according to the constant-acceleration kinematic equations.

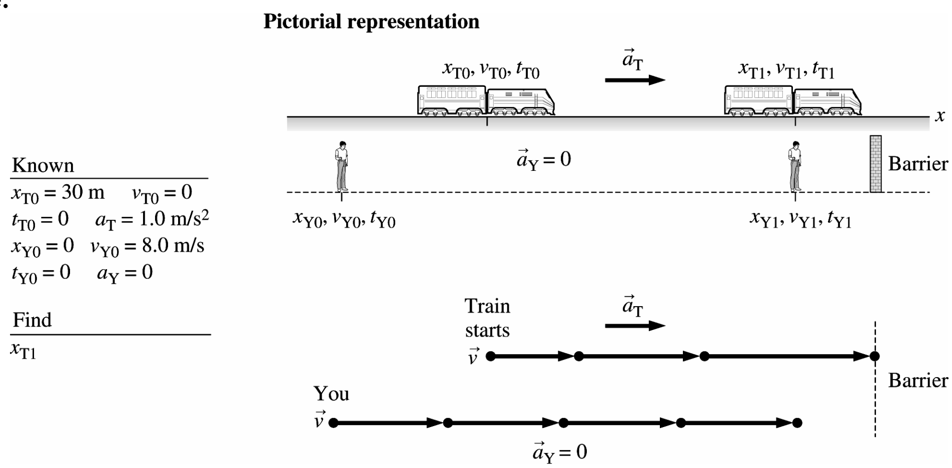
Visualize:



Solve: Using the known values, we have

$$v_1^2 = v_0^2 + 2a_0(y_1 - y_0) \Rightarrow (-10 \text{ m/s})^2 = v_0^2 + 2(-9.8 \text{ m/s}^2)(5.0 \text{ m} - 0 \text{ m}) \Rightarrow v_0 = 14 \text{ m/s}$$

2.68. Model: We use the particle model for the large train and the constant-acceleration equations of motion.
Visualize:



Solve: Your position after time t_{Y1} is

$$x_{Y1} = x_{Y0} + v_{Y0}(t_{Y1} - t_{Y0}) + \frac{1}{2}a_Y(t_{Y1} - t_{Y0})^2$$
$$= 0 \text{ m} + (8.0 \text{ m/s})(t_{Y1} - 0 \text{ s}) + 0 \text{ m} = 8.0\ t_{Y1}$$

The position of the train, on the other hand, after time t_{T1} is

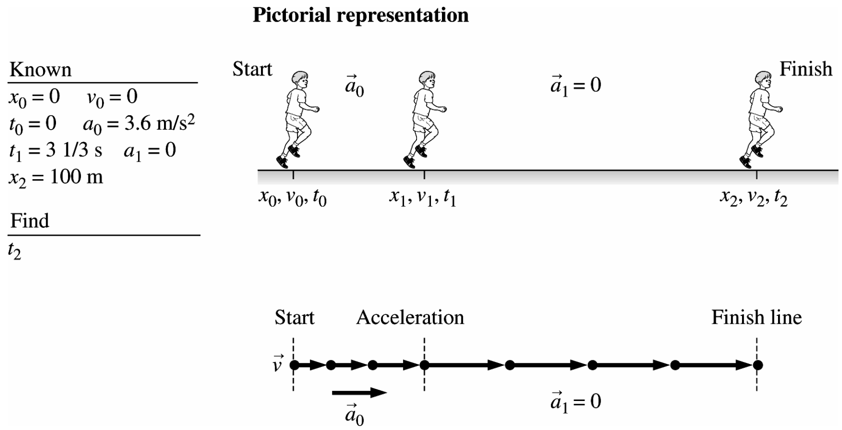
$$x_{T1} = x_{T0} + v_{T0}(t_{T1} - t_{T0})^2 + \frac{1}{2}a_T(t_{T1} - t_{T0})^2$$
$$= 30 \text{ m} + 0 \text{ m} + \frac{1}{2}(1.0 \text{ m/s}^2)(t_{T1})^2 = 30 + 0.5t_{T1}^2$$

The two positions x_{Y1} and x_{T1} will be equal at time $t_{Y1}(=t_{T1})$ if you are able to jump on the back step of the train. That is,

$$30 + 0.5t_{Y1}^2 = 8.0t_{Y1} \Rightarrow t_{Y1}^2 - (16 \text{ s})t_{Y1} + 60 \text{ s}^2 = 0 \Rightarrow t_{Y1} = 6 \text{ s and } 10 \text{ s}$$

The first time, 6 s, is when you will overtake the train. If you continue to run alongside, the accelerating train will then pass you at 10 s. Let us now see if the first time $t_{Y1} = 6.0$ s corresponds to a distance before the barrier. From the position equation for you, $x_{Y1} = (8.0 \text{ m/s})(6.0 \text{ s}) = 48.0$ m. The position equation for the train will yield the same number. Since the barrier is at a distance of 50 m from your initial position, you can just catch the train before crashing into the barrier.

2.79. Model: Use the particle model.
Visualize:



Solve: **(a)** Substituting into the constant-acceleration kinematic equation

$$x_2 = x_1 + v_1(t_2 - t_1) + \frac{1}{2}a_1(t_2 - t_1)^2 \Rightarrow 100\text{ m} = x_1 + v_1\left(t_2 - \frac{10}{3}\right) + 0\text{ m}$$

$$t_2 = \frac{100 - x_1}{v_1} + \frac{10}{3}$$

Let us now find v_1 and x_1 as follows:

$$v_1 = v_0 + a_0(t_1 - t_0) = 0\text{ m/s} + (3.6\text{ m/s}^2)\left(\frac{10}{3}\text{ s} - 0\text{ s}\right) = 12\text{ m/s}$$

$$x_1 = x_0 + v_0(t_1 - t_0) + \frac{1}{2}a_0(t_1 - t_0)^2 = 0\text{ m} + 0\text{ m} + \frac{1}{2}(3.6\text{ m/s}^2)\left(\frac{10}{3}\text{ s} - 0\text{ s}\right)^2 = 20\text{ m}$$

The expression for t_2 can now be solved as

$$t_2 = \frac{100\text{ m} - 20\text{ m}}{12\text{ m/s}} + \frac{10\text{ s}}{3} = 10\text{ s}$$

(b) The top speed = 12 m/s which means $v_1 = 12$ m/s. To find the acceleration so that the sprinter can run the 100-meter dash in 9.9 s, we use

$$v_1 = v_0 + a_0(t_1 - t_0) \Rightarrow 12\text{ m/s} = 0\text{ m/s} + a_0 t_1 \Rightarrow t_1 = \frac{12\text{ m/s}}{a_0}$$

$$x_1 = x_0 + v_0(t_1 - t_0) + \frac{1}{2}a_0(t_1 - t_0)^2 = 0\text{ m} + 0\text{ m} + \frac{1}{2}a_0 t_1^2 = \frac{1}{2}a_0 t_1^2$$

Since $x_2 = x_1 + v_1(t_2 - t_1) + \frac{1}{2}a_1(t_2 - t_1)^2$, we get

$$100\text{ m} = \frac{1}{2}a_0 t_1^2 + (12\text{ m/s})\ (9.9\text{ s} - t_1) + 0\text{ m}$$

Substituting the above equation for t_1 in this equation,

$$100\text{ m} = \left(\frac{1}{2}\right)a_0\left(\frac{12\text{ m/s}}{a_0}\right)^2 + (12\text{ m/s})\left(9.9\text{ s} - \frac{12\text{ m/s}}{a_0}\right) \Rightarrow a_0 = 3.8\text{ m/s}^2$$

(c) We see from parts (a) and (b) that the acceleration has to be increased from 3.6 m/s^2 to 3.8 m/s^2 for the sprint time to be reduced from 10 s to 9.9 s, that is, by 1%. This decrease of time by 1% corresponds to an increase of acceleration by

$$\frac{3.8-3.6}{3.6}\times 100\% = 5.6\%$$