For a floating body the magnitude of the buoyant force F_b must equal the weight of the body F_g, or:

$$F_b = F_g \qquad \text{(eq. 1)}$$

which can be written as:

$$m_{liquid}\, g = m_{object}\, g \qquad \text{(eq. 2)}$$

where $\mathbf{m_{liquid}}$ is the mass of the liquid displaced by the floating object and $\mathbf{m_{body}}$ is the mass of the floating object. **g** denotes the acceleration due to the Earth's gravity.

Before you read on*, I want you to think about the idea of an equilibrium condition:*

- *What does it mean when both forces are equal in magnitude, but opposite directions?*
- *What happens to the object in this case?*
- *What would happen if F_g is larger than F_b ($F_g > F_b$).*
- *What would happen if F_g is smaller than F_b ($F_g < F_b$).*
- *Can you imagine real life examples for these two cases?*

We can replace the masses by the densities ρ and volumes V. With ρ = m / V we can write eq. 2 as:

$$\rho_{liquid} V_{liquid}\, g = \rho_{object} V_{object}\, g \qquad \text{(eq. 3)}$$

assuming objects having constant cross-sections A we can write their volume as the product of their height and cross-sectional area A, or V = h A. Substituting in eq. 3 and canceling terms that appear on both sides of the equation we get:

$$\rho_{liquid}\, h_l = \rho_{object}\, h_{object} \qquad \text{(eq. 4)}$$

To derive equation 4 we used the definition of density ρ = m / V.

- *Why is this definition useful and what does it mean?*
- *What are the units of density?*
- *If a body has a density of 5 g/cm^3, and a volume V = 3 cm, what is its mass?*
- *How will its mass change if you double (triple) the volume?*
- *If you draw a graph of mass vs. volume, what would it look like?* ***YES, do it, draw it carefully, and do it now!***

Now consider three columns of equal cross-sectional areas. Column 1 cuts through object number 1 and through the liquid below, column 2 cuts only through water, and column 3 cuts through object number 2 and the water below, as indicated in the figure below. Since the masses of the fluid displaced by objects 1 and 2 equal the masses of the two objects we can use equation 4 write the following relationships (refer to Fig. 1 for variables and indices used):

$$h_1 \rho_1 A + l_1 \rho_l A = h_2 \rho_2 A + l_2 \rho_l A = x \rho_l A \quad \text{(eq. 5)}$$

(In the equation above we did not cancel out the cross-sectional area A, though we could, since we specified that A is the same for each column.)

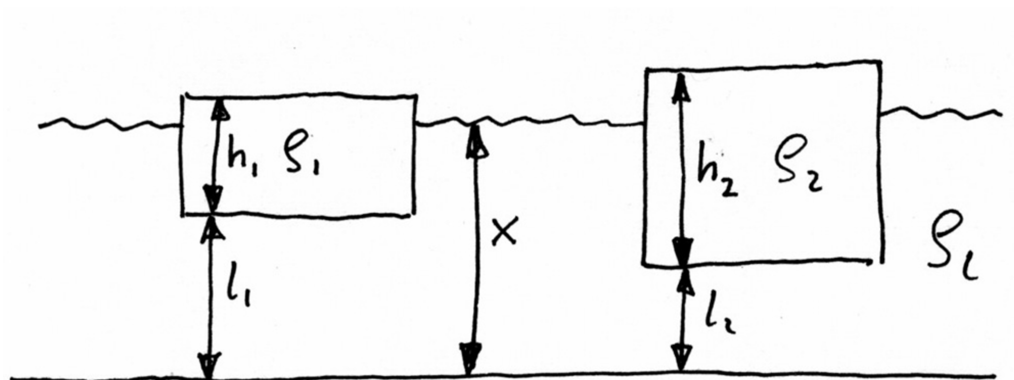

Fig. 1

***Before you return to the isostasy problem** you should think about the meaning of equation 5 and how we can use it in geology. Why can we write this equation, and is it really necessary to know the value of x. For geologists this question translates into: Do we have to know the thickness of the mantle, or can we assume some (arbitrary) value for x as long as ... (what?).*